

Warranty Analysis of Repairable Systems

David Trindade and Jeff Glosup, Sun Microsystems
Bill Heavlin, Google

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Repairable Systems

- Main focus is repairable systems
 - > A system is repairable if by replacing or repairing system components, the system can be returned to service.
 - > After the repair, the system may be
 - > As good as new (renewal process)
 - > Worse than new 😞
 - > Better than new 😊
 - > Behavior of system after repair depends on repair actions and effects of replaced components.
 - > A key characteristic of a repairable system is that an individual system may fail multiple times for the same or different causes.

Warranty and Reliability Analysis

- Warranty analysis and reliability analysis of field data are closely related areas.
- The authors have used graphical Time Dependent Reliability (*TDR*) methods extensively for analyzing product field reliability data.
 - > *TDR* is for use with repairable systems.
- *TDR* methods are also very useful for the analysis of warranty data.
 - > Can isolate specific failure modes and associated cost data easily

Time Dependent Reliability

- Graphical technique based on studying the cumulative occurrence of events of interest over time on individual systems
 - > The overall behavior is represented by the average of events across systems at fixed times – the mean cumulative function (*MCF*).
 - > Can change definition of events of interest to suit study
 - > Can be all field failures for field reliability
 - > Can be warranty claims for warranty data analysis
 - > Can be costs of repairs or claims
 - > *TDR* approach is consistent with warranty analysis techniques such as suggested by Kalbfleisch, Lawless and Robinson.

Estimating the *MCF*

- Consider a collection of M observed times of events t_k for a set of units sold since a start time t_0
 - > Assume the event times are set in increasing order so $t_0 \leq t_1 \leq t_2 \leq \dots \leq t_M$
 - > Note times may be calendar dates or may represent time in operation, e.g., system age
 - > The number of units active at any time is variable due to manufacture and sale at different times.
 - > Let $N(t_k)$ be the number of units active at the time represented by t_k

Estimating the *MCF* - 2

- The Mean Cumulative Function (*MCF*) is defined successively as

$$MCF(t_0) = 0$$

$$MCF(t_k) = MCF(t_{k-1}) + \frac{w(t_k)}{N(t_k)}$$

where $w(t_k)$ is an incremental weight

- Incremental weight $w(t_k)$ may be
 - > Number of failures at time t_k
 - > Costs of failures at time t_k

Extending the Analysis

- The *TDR* technique can be used to graphically explore the modes of failure by re-defining the events of interest.
 - > For example, we can produce a MCF for each failure mode or failing component.
 - > The resulting chart is a ***dynamic Pareto chart*** of failing components.
 - > Each of these curves should fall beneath the overall MCF curve.
 - > These component curves should add up to produce the overall MCF curve.
 - > Again, cost data can be incorporated by weighting the individual component fails by their repair cost – scaled in dollars.

Example

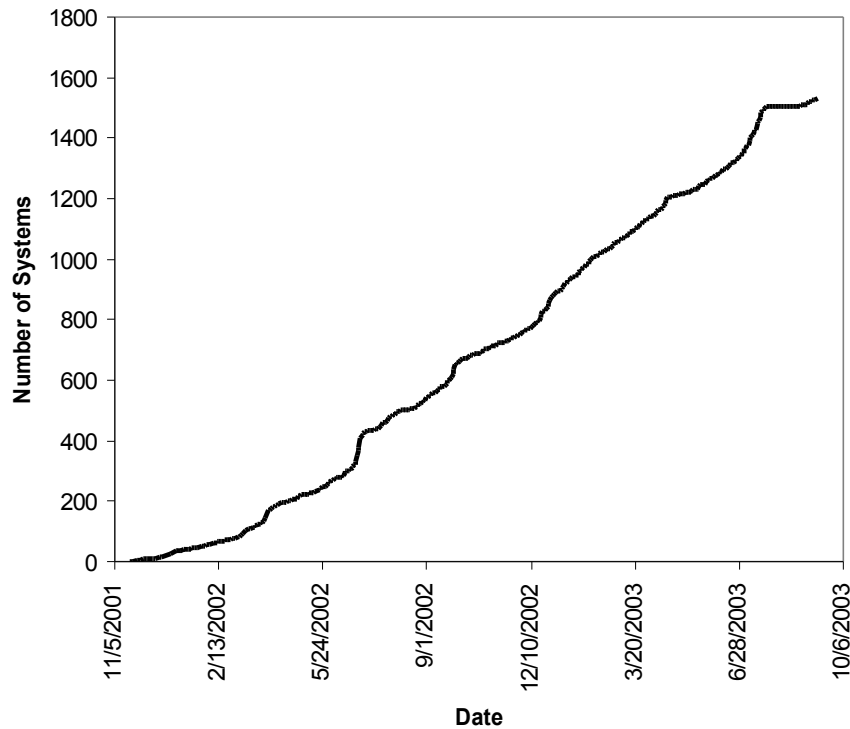
- This example contains roughly 1500 systems manufactured and installed over approximately a two year period.
- The data represent all systems manufactured over this period.
- Each system consists of many components, but four of these components are responsible for a large fraction of the failures.
- Adjustments were made for missing data. (See Glosup, 2002)

Age Versus Date Dependency

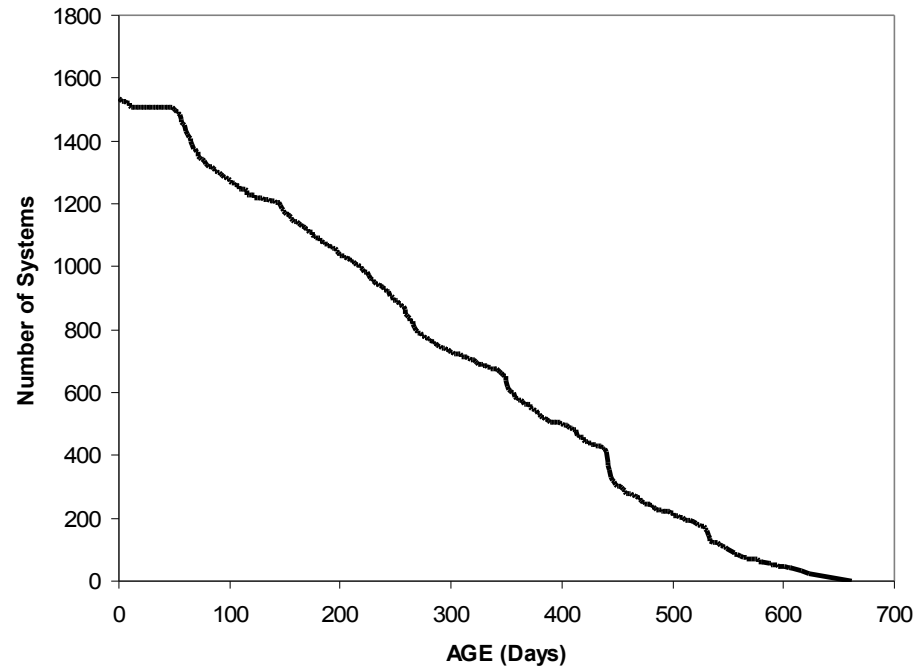
- In *TDR* analysis, we typically view the data in two forms.
- ***Age dependency*** looks at failure causes related to the age of the system.
- ***Date dependency*** shows causes related to events associated with a specific date. Possible causes include software updates, general maintenance actions, physical relocation of systems, sudden environmental changes, and so on.

Active Systems Plots

Active Systems Versus Date

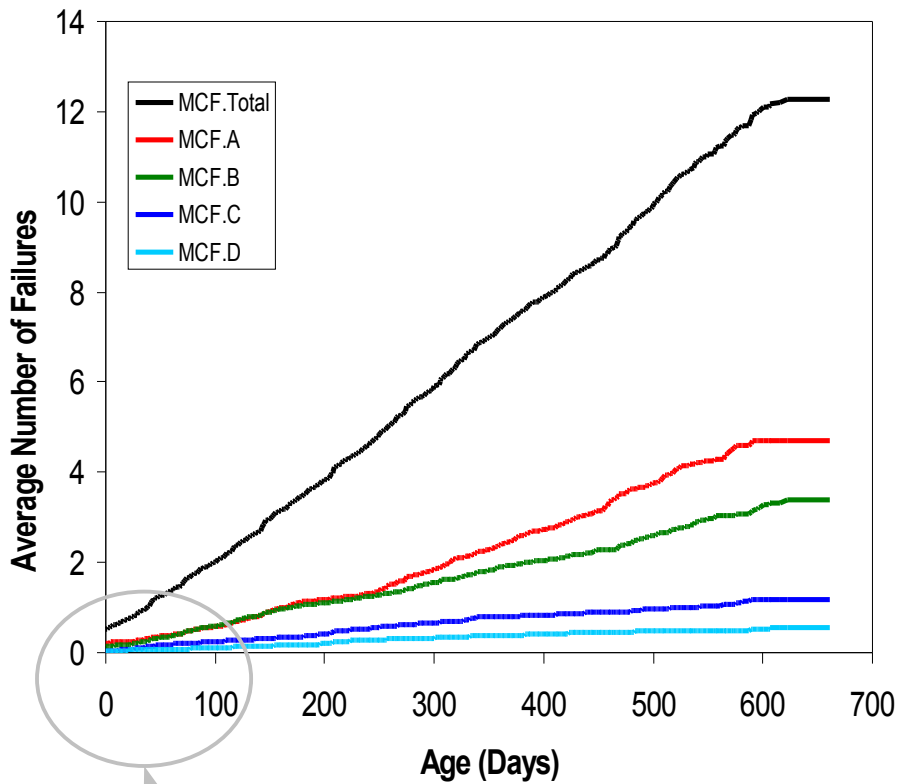


Active Systems Versus Age

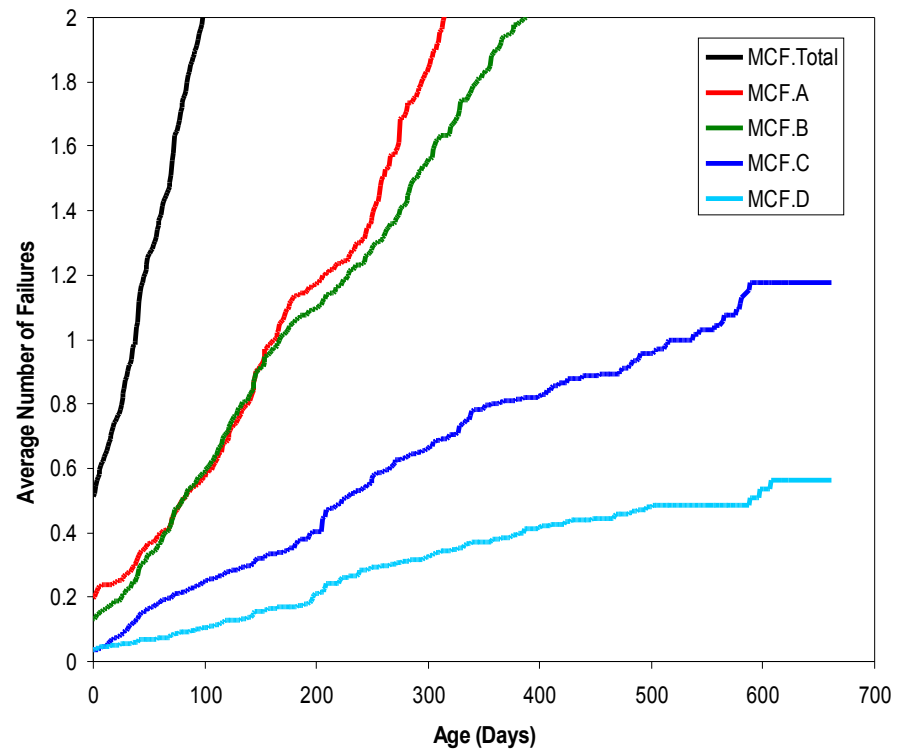


MCF Plots by Age

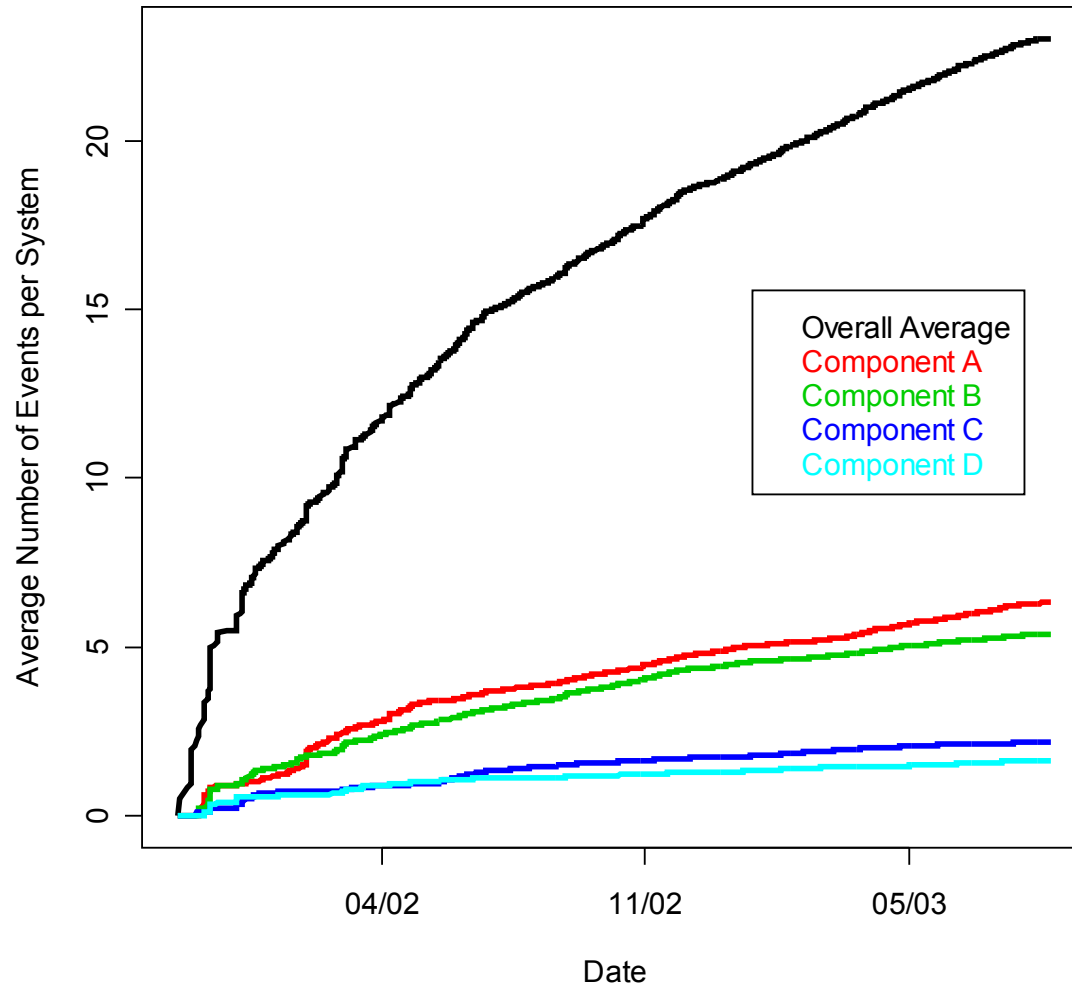
Total MCF and Failure Mode Specific MCF Plots



Total MCF and Failure Mode Specific MCF Plots



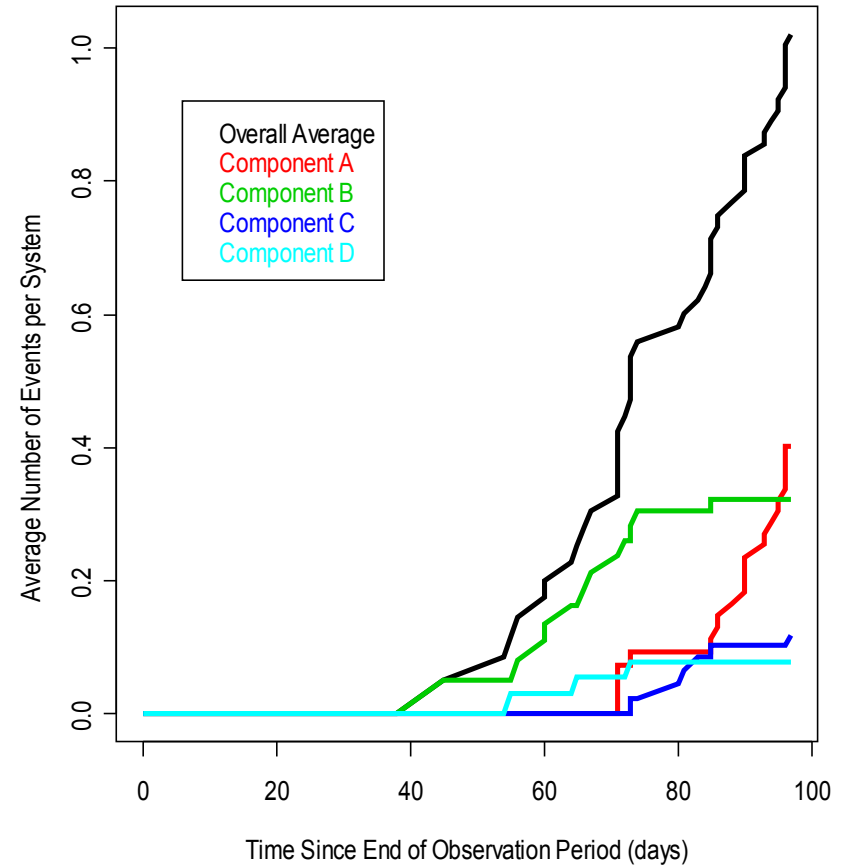
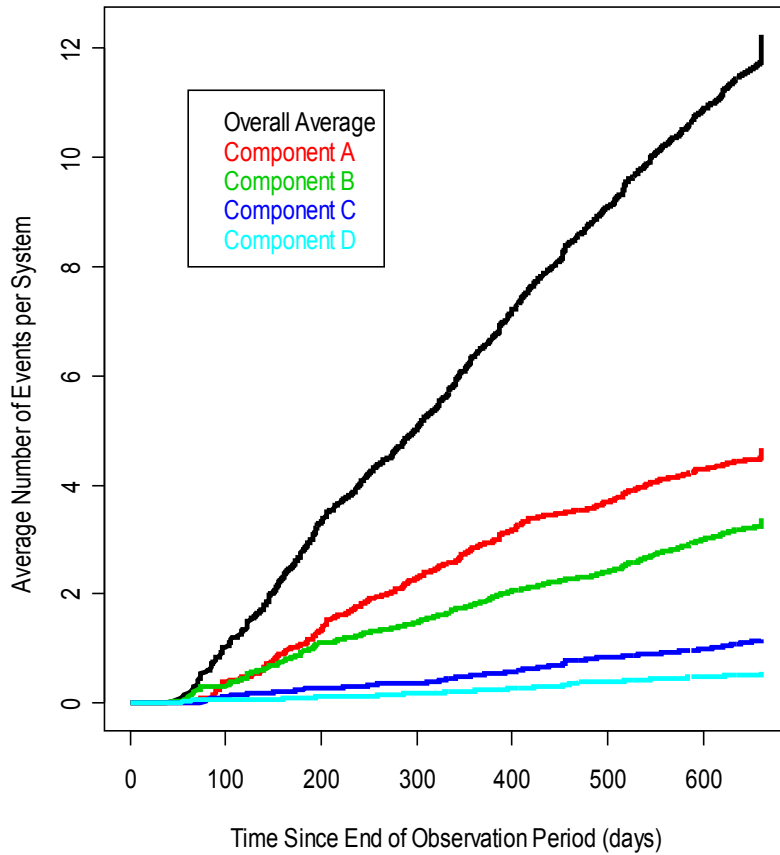
MCF Plots by Date



MCF Plot with Reversed Time

- The *MCF* plot can also be reprotayed with the time axis reversed and the *MCF* curves shifted.
 - > Shows most recent events to the left side of the graph.
 - > Graph emphasizes to a greater extent the differences in failure modes in the later ages associated with the most recent events.
 - > Reference time zero point used is oldest age.

Reverse Time Plots



NHPP Power Law Model for *MCF* Vs. Age

- Power law model for MCF:

$$M(t) = \alpha t^{\beta}$$

- Intensity function (theoretical recurrence rate)

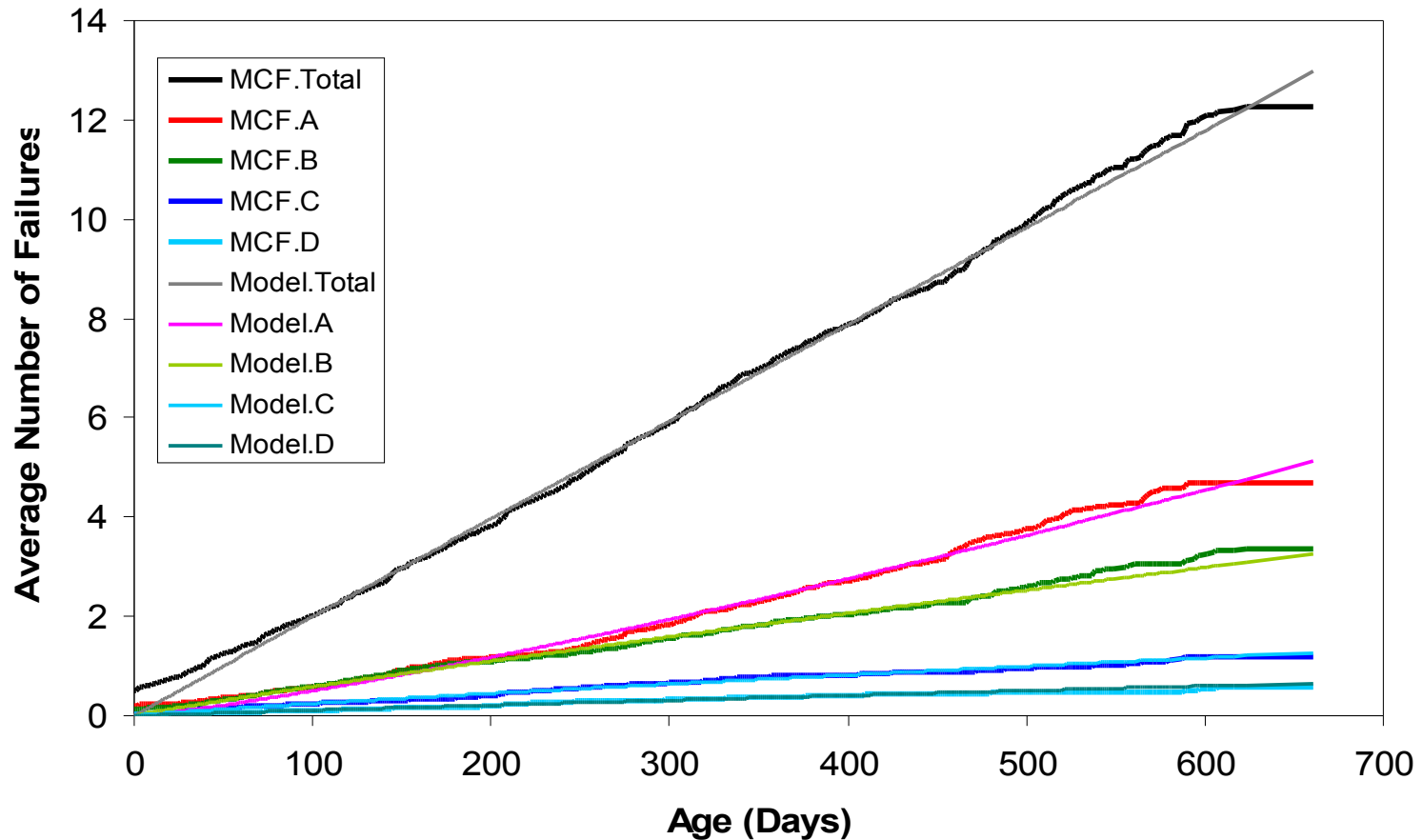
$$\lambda(t) = dM(t)/dt = \alpha\beta t^{\beta-1}$$

Parameter Estimation for Power Law Model

- Although *MLE* methods exist for estimating the parameters of the Power Law model (see Crow, 1974), a simple and direct approach is to estimate the parameters by minimizing the sum of squares of the residuals between the *MCF* and the model.
- The non-linear least squares **Solver** routine in the spreadsheet program EXCEL easily facilitates the estimation.

Power Law Model Fits

Total MCF and Failure Mode Specific MCFs with Model Fits



Parameters of Power Law Model

	α	β
• Overall MCF	.0207	.991
• Component A	.00175	1.23
• Component B	.00887	.910
• Component C	.00455	.868
• Component D	.00148	.940

Projections Using the Power Law Model

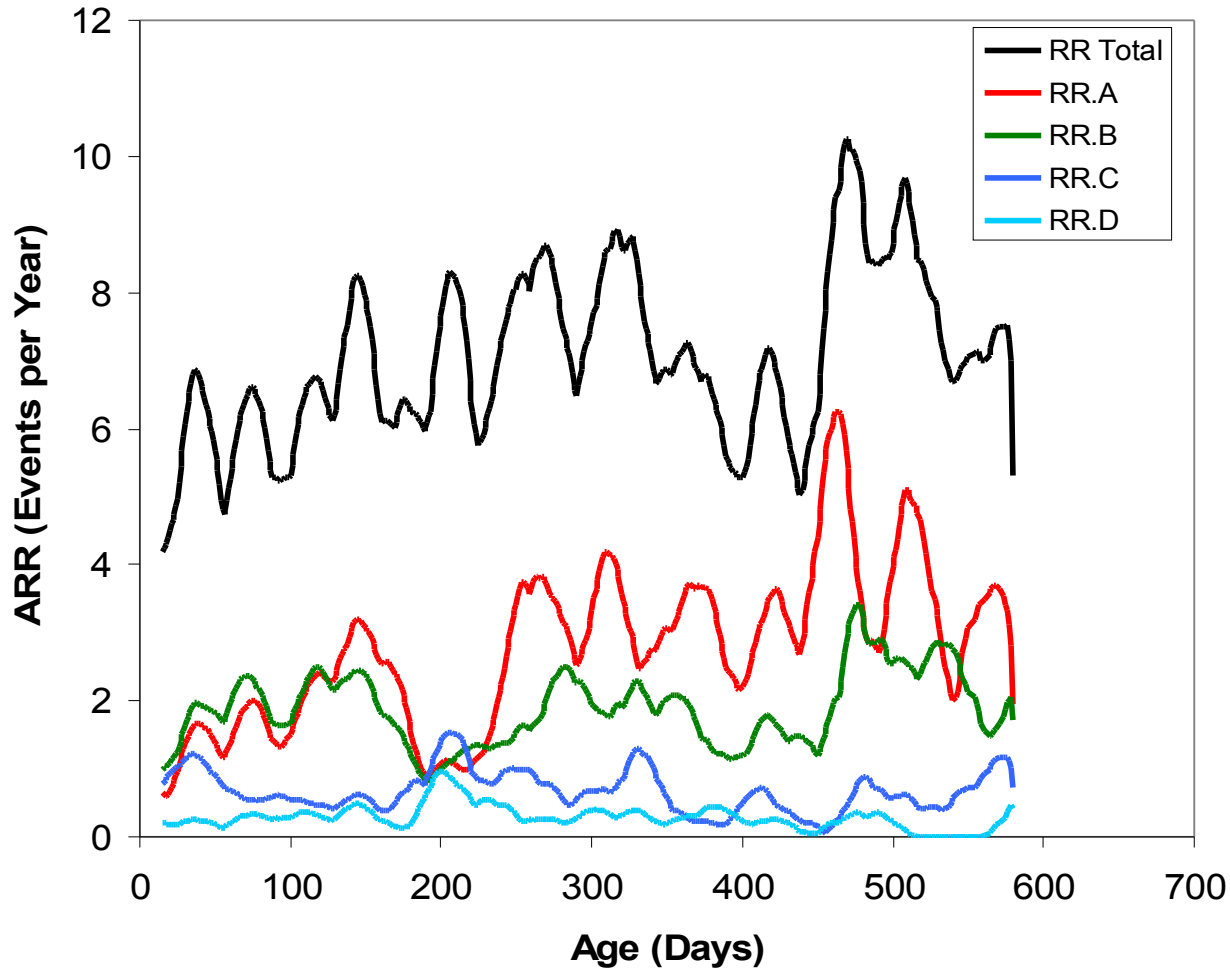
- With the estimated parameters, it is possible to use models to do overall or failure mode specific projections for future ages or dates.
- Such projections are a key component of warranty analysis, for example, to estimate spare parts inventories and staffing levels for field support.

Empirical Recurrence Rates (*RR*)

- It is possible to numerically differentiate the *MCF* curve and determine recurrence rates as a function of the system age or calendar date.
- A simple approach to the recurrence rate calculations is possible using the built-in spreadsheet SLOPE function.
- A slope is found for a window (odd) number of consecutive *MCF* points and plotted at the median age of the points. Then, the first point is dropped and another consecutive point added, with the process repeated. (See Trindade, 1975)

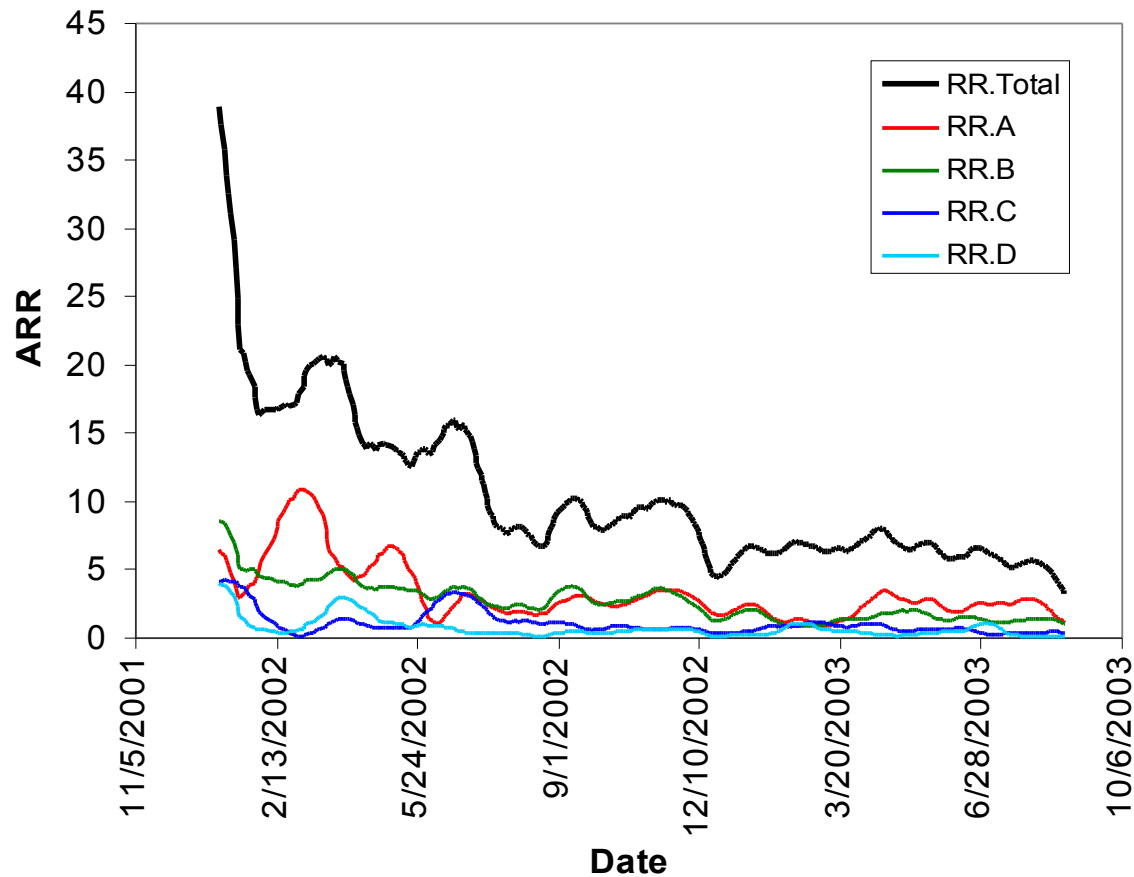
ARR (Empirical) Vs. Age

Annualized Recurrence Rate (ARR) Vs. Age
(31 Day Window)



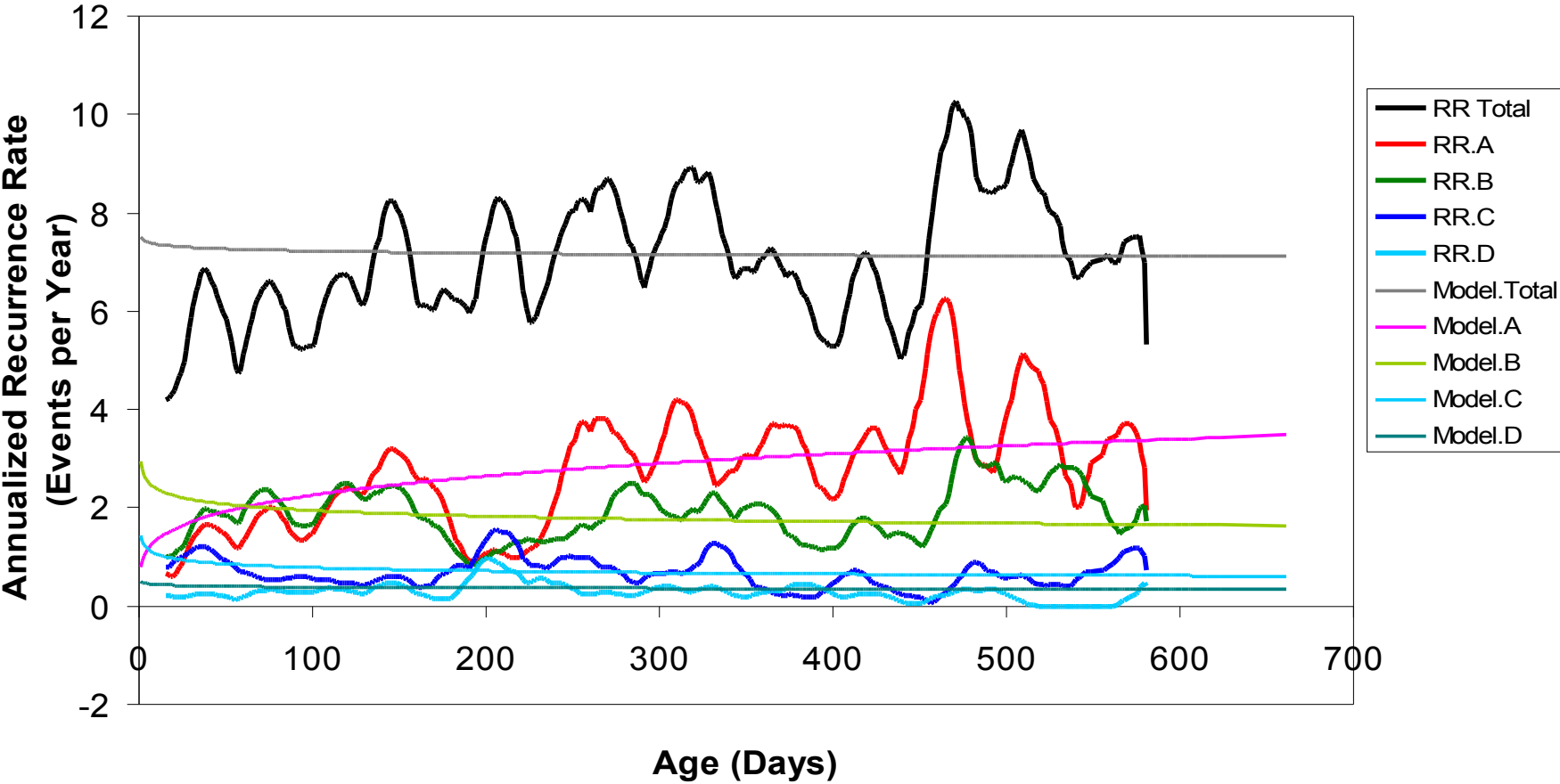
Annualized Empirical Recurrence Rates (ARR) Vs. Date

Annualized Recurrence Rate (ARR) Vs. Date
31 Day Window



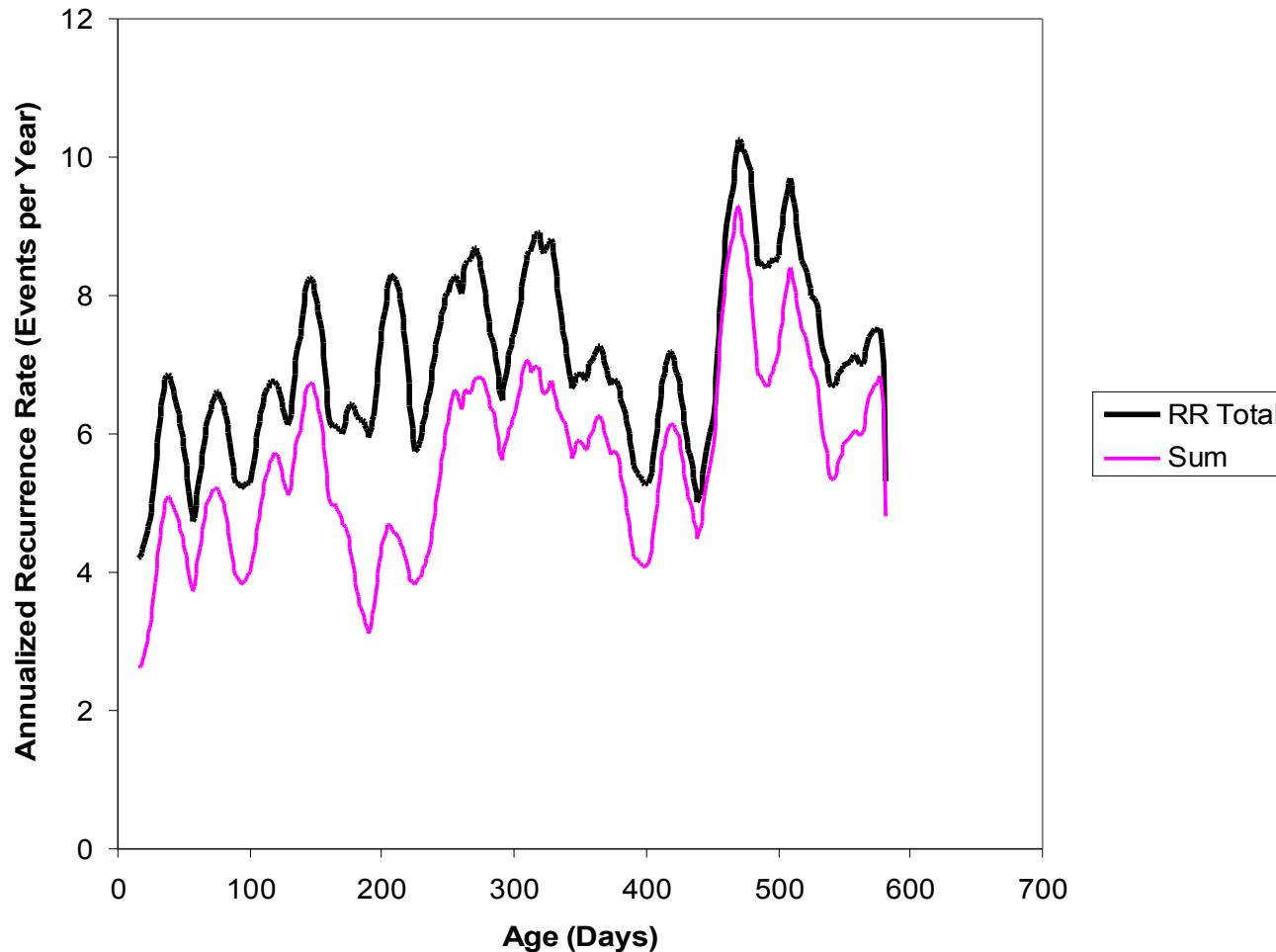
Annualized Recurrence Rates and Power Law Model Fits

Total ARR and Failure Mode Specific ARR with Model Fits



Total RR Vs. Failure Mode RR Sum

Recurrence Rate Total and Sum of Four Modes

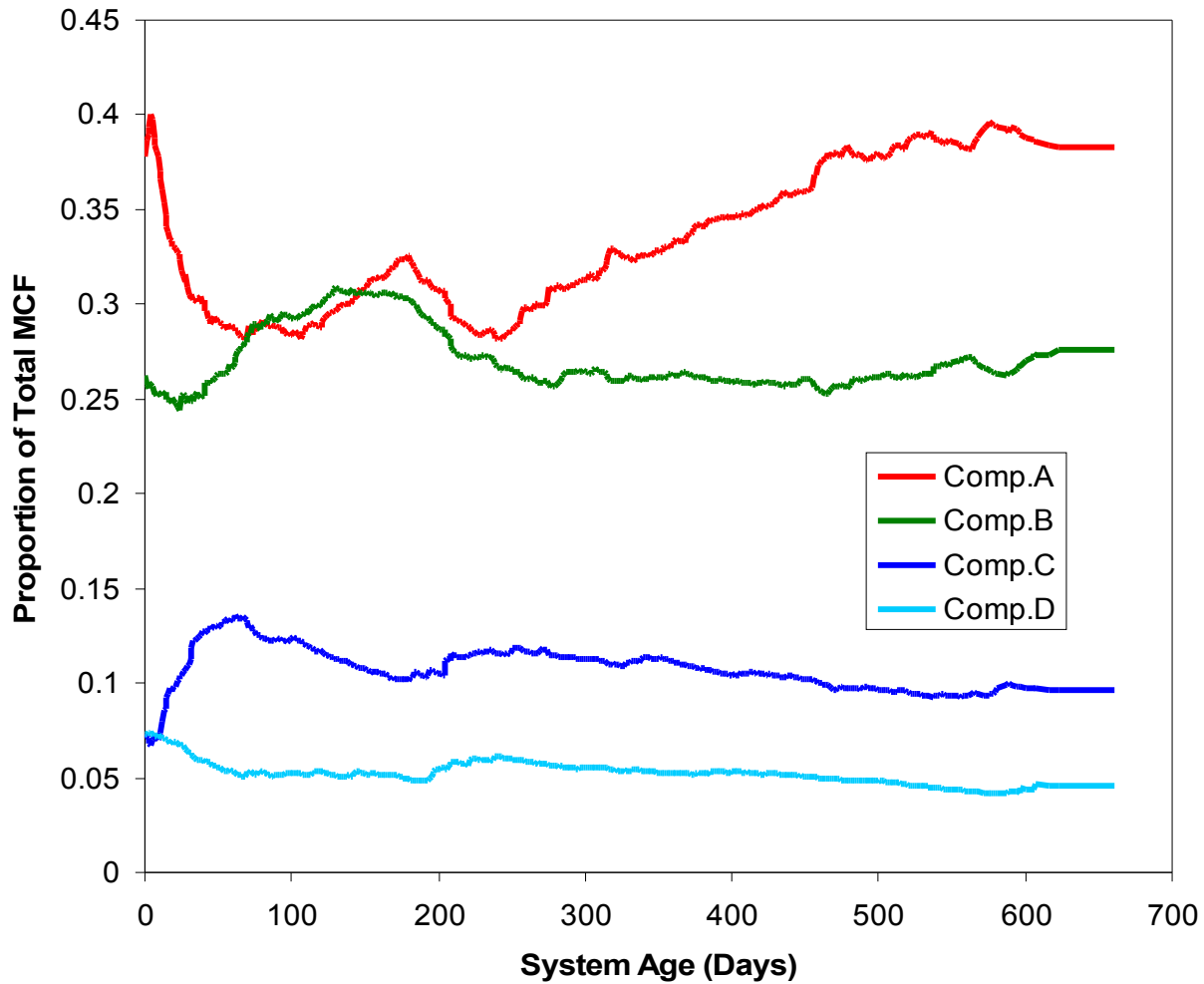


Dynamic Pareto

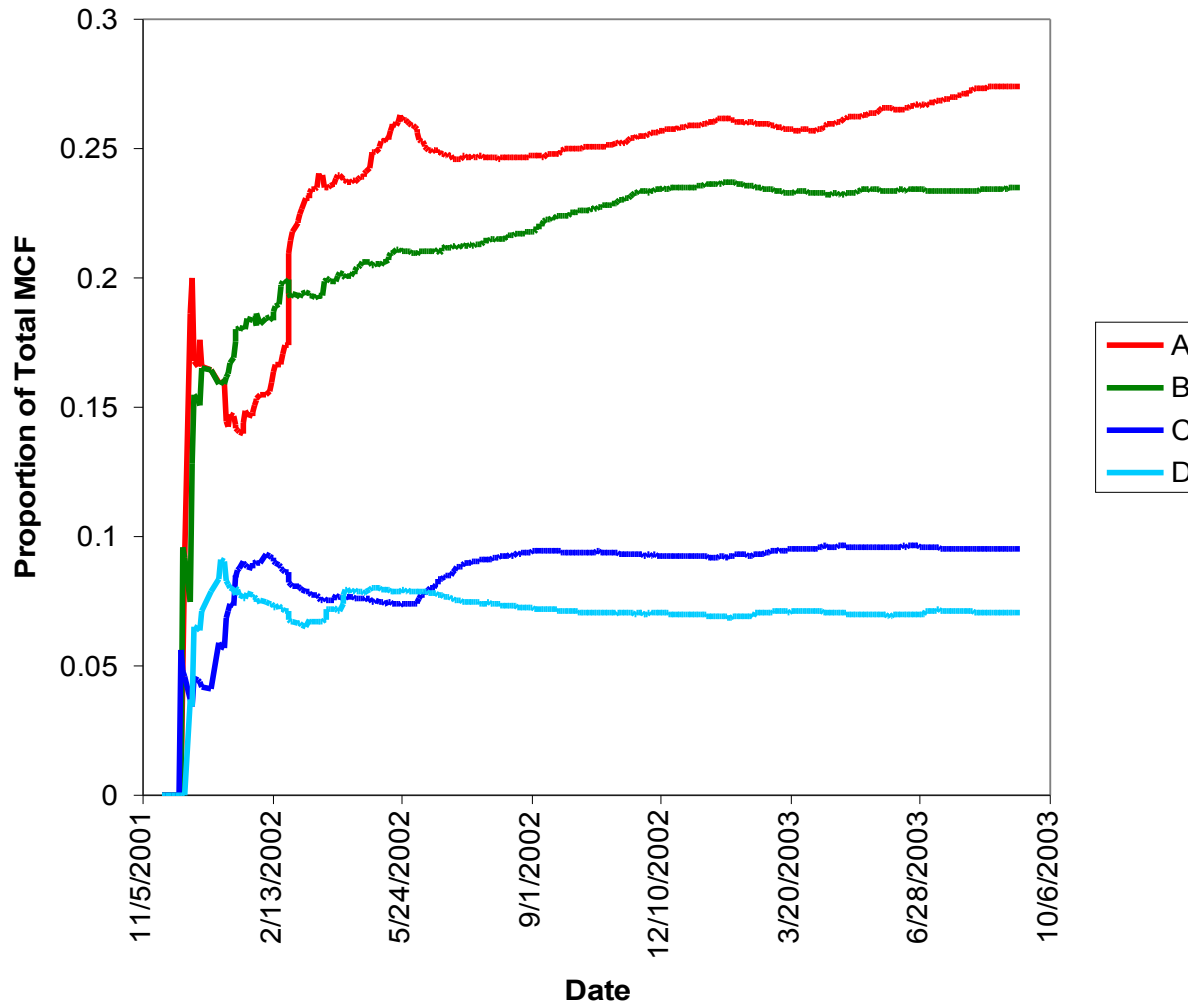
- The idea of the **dynamic Pareto** can be made easier to interpret by rescaling the curves.
- For the components, take the ratio of the component *MCF* to the overall *MCF*, e.g, for component *A*

$$R_A(t_k) = \frac{MCF_A(t_k)}{MCF_{Total}(t_k)}$$

Dynamic MCF Pareto by Age

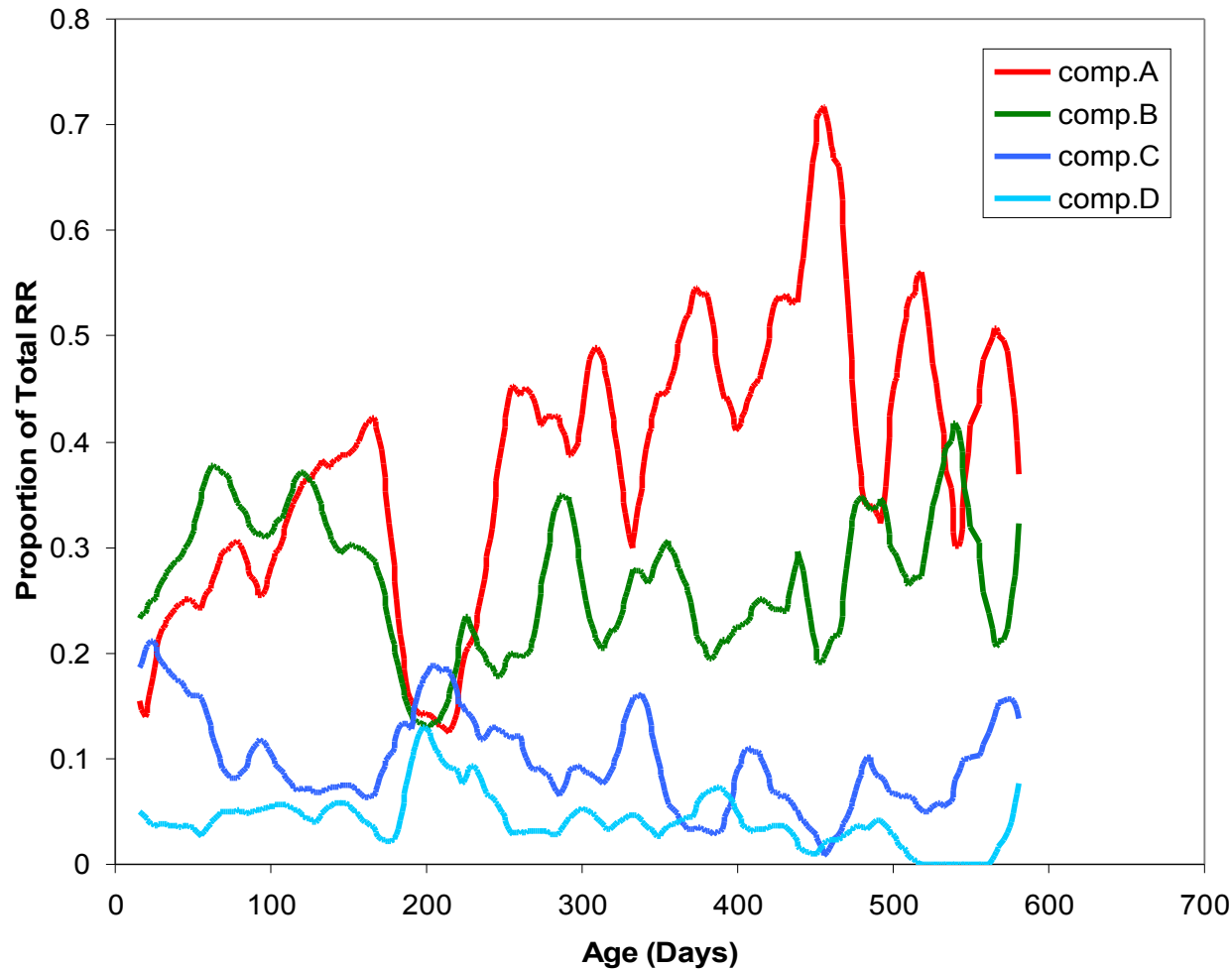


Dynamic MCF Pareto by Date

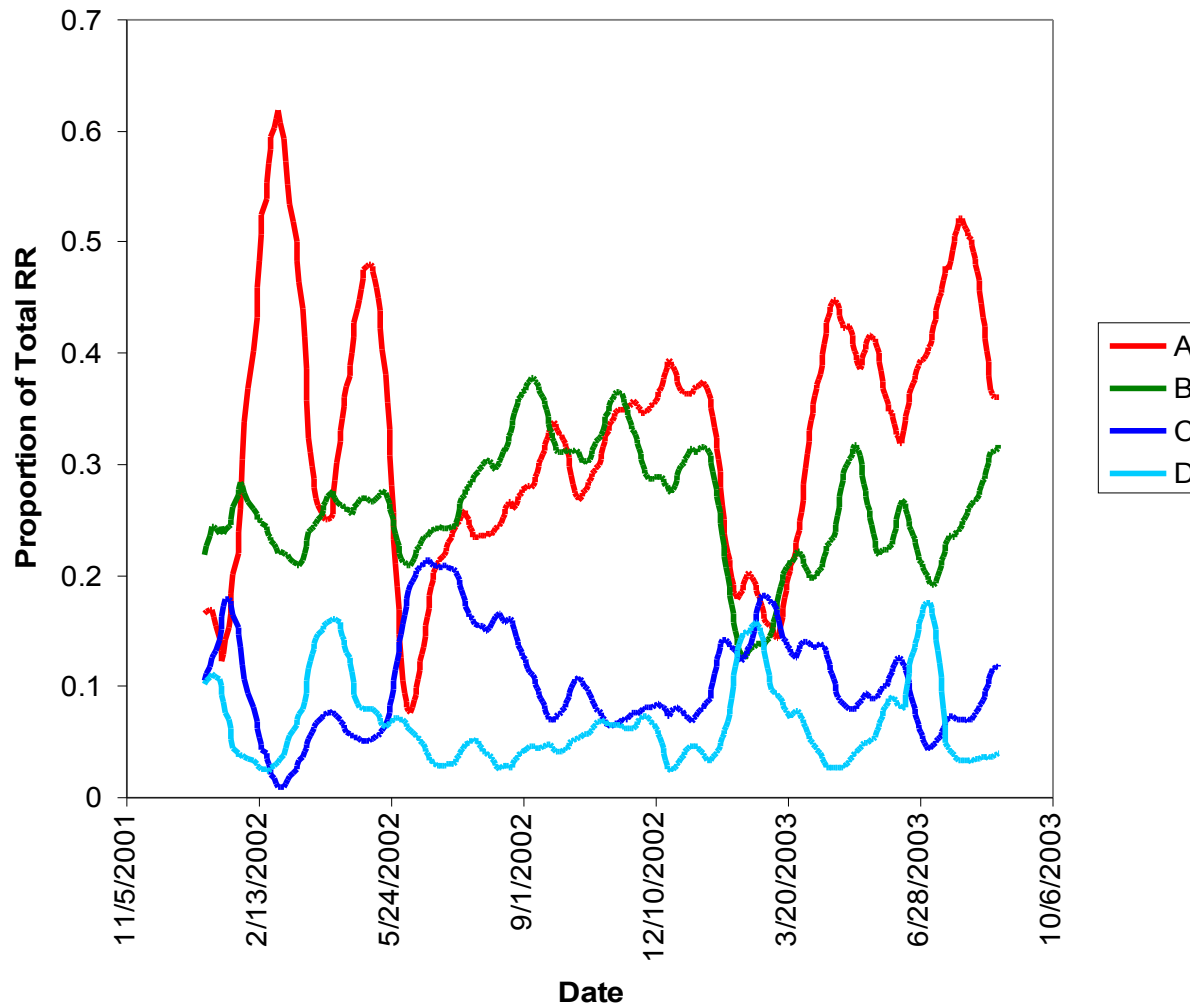


Dynamic RR Pareto by Age

Dynamic Pareto Empirical Recurrence Rate Plots



Dynamic RR Pareto by Date



Average Cost per Unit Time

- The Cost per Unit Time computed from the *MCF*:

$$MCF(t_0) = 0$$

$$MCF(t_k) = MCF(t_{k-1}) + \frac{\sum_m c_m n(m, t_k)}{N(t_k)}$$

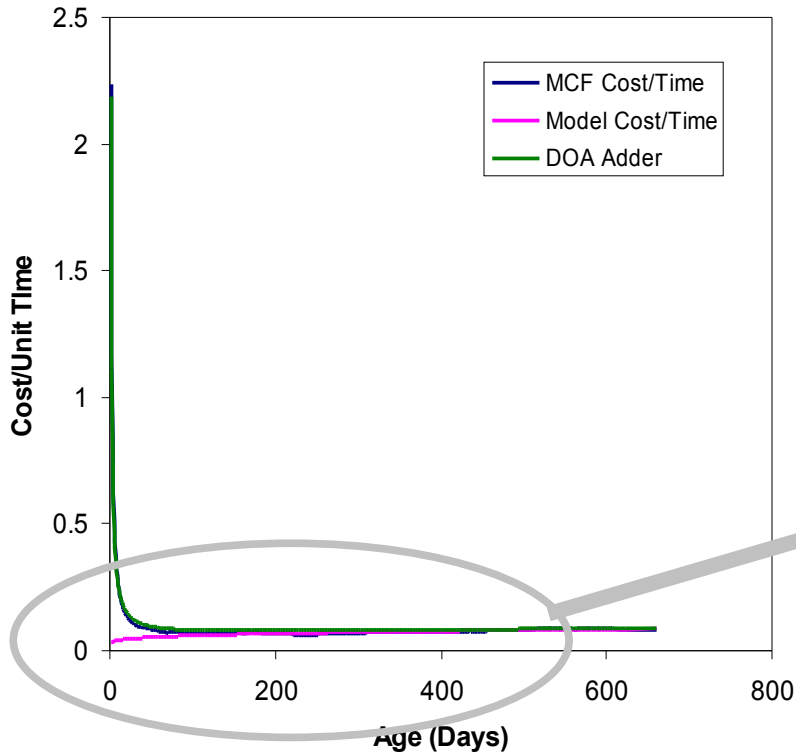
Cost per Unit Time = $MCF(T)/(T - t_0)$, where

$n(m, t_k)$ = number of fails of failure mode m at time t_k ,

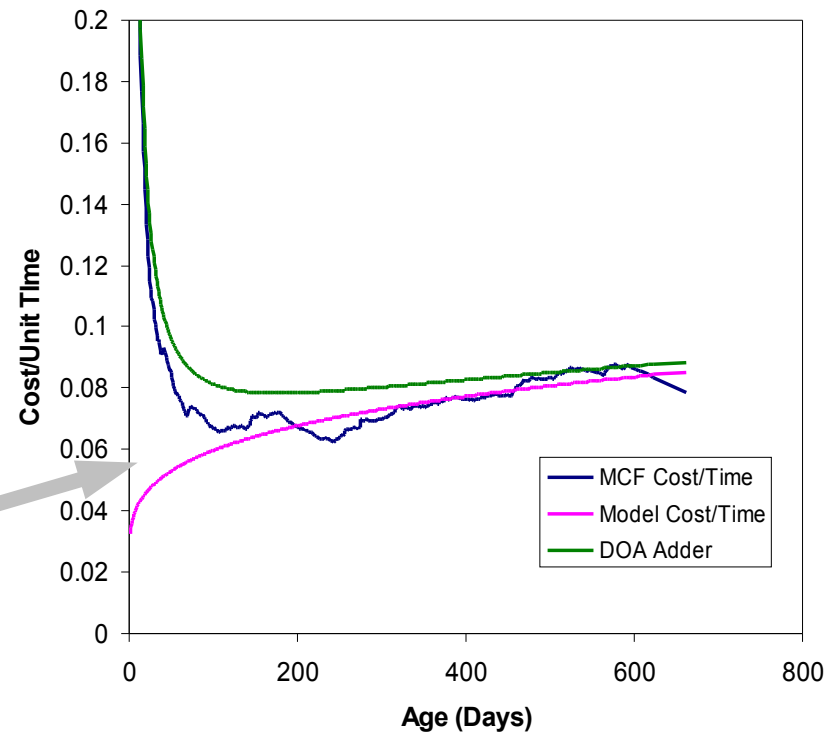
c_m = (marginal) cost of repairing failure mode m .

Cost Per Unit Time Plots

Costs Per Unit Time



Costs Per Unit Time



Assigned Cost Weights: A(10), B(1), C(1), D(1)

MCF Analysis Benefits

- MCFs are close to the data and so one can do data-sensitive calculations.
- Benefits shows up with:
 - > (a) the failure-mode-specific MCFs
 - > (b) calendar MCFs
 - > (c) reverse-time/recent failure plotting
 - > (d) the dynamic Pareto plots
- MCFs work with large populations & MCFs work with small populations
 - > The empirical approach is revealing with cost-per-unit-time plots, too, and goodness-of-fit plots of power law models.

Dynamic Pareto and Visualization

- The dynamic Pareto concept is very useful in the analysis of warranty claims.
- Both MCFs and RRs can reveal interesting aspects of the data using the dynamic Pareto approach.
- These visualization methods are easy to grasp and provide excellent insight into specific failure causes that vary with age or date.
- In most cases, the analysis can be easily accomplished using simple spreadsheet programs.
- Power law model complements *MCF* & *RR* analysis.

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