COMPONENTS OF VARIANCE AND NESTED FACTORIAL DESIGNS

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Abstract: A common objective in the semiconductor industry is process improvement by the reduction of variation. Consequently, identification of the process related factors that influence variability is important. Of special consideration in integrated circuit (IC) manufacturing is the nested nature of many variables. For example, chips or dies are nested in wafers which are in turn nested in lots. To focus process improvement efforts, an engineer might conduct a components of variance study. Unfortunately, failure to distinguish between fixed and random effects can lead to negative components and misinterpretation of results. In this paper, we describe the problem and discuss possible methods of resolution.

1. Introduction

An engineer wanted to investigate a new technology: X-ray lithography. The objective was to understand the relative importance of various sources of nonuniformity affecting critical dimensions. An designed experiment was conducted which involved extensive nesting of factors. There were four wafers from one lot. Within each wafer there were sixteen fields. Each field contained three columns. Each column consisted of three sites (rows). Two measurements were performed on each site. Thus, the sources of variation were wafer, field, column, site, and measurement The engineer analyzed the data using components of variance techniques. The interest centered on determining the contribution of each component to total variation. The information would help identify which areas of the lithography process needed attention to reduce nonuniformity.

The analysis produced a negative component of variance associated with field. See Table 1 below.

Table 1. Analysis of Variance

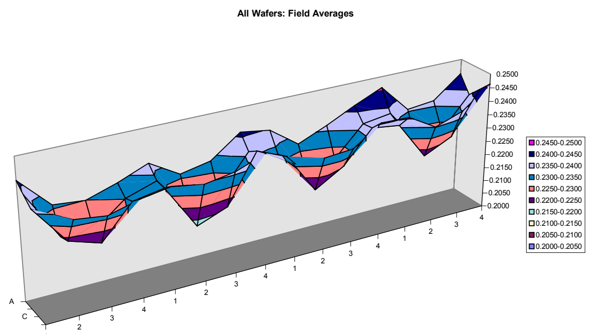
X-Ray Photolithography Study

ANOVA (Components of Variance)

SS	0.0043	0.0520	0.1244	0.0492	0.0018
DF	3	60	128	384	576
MS	0.001446	0.000867	0.000972	0.000128	3.2E-06
F Ratio	1.67	0.89	7.59	40.04	
Significance	18.34%	68.60%	0.00%	1.1E-280	
	Wafer	***Field***	***Column***	***Site***	***Test***
Sum DF	4	64	192	576	
Com Var	2.01E-06	-5.83E-06	1.41E-04	6.24E-05	3.20E-06
%	1.0%	-2.8%	67.5%	30.0%	1.5%

The standard statistical procedure is to assume that a negative component is negligible and can be set equal to zero. However, the engineer refused to accept the interpretation, feeling that the "negligible" components indeed accounted for significant variation. A three-dimensional view of the field to field variation for all four wafers is show as Figure 1.

Figure 1. Field to Field Variation for Four Wafers



2. Analysis

In a components of variance study, the levels of all factors are a sample of all possible levels of interest. Factors are said to have random effects. A nested or hierarchical design can be used to investigate components of variance. The experimental error and factor effects are assumed statistically independent, normally distributes random variables with zero means but possibly different standard deviations. A nested design can have both fixed and random factor effects.

The variance of the response is the sum of the individual variances of the main effects, interaction effects, and uncontrolled error. The random effects contribute to the overall variation observable in the response variable, but not to differences in factor- level means. If all factor levels are random with zero expected mean, then we have a components of variance model.

In a nested design, levels of one factor differ with each level of another factor. Randomization within a factor level is possible. No interaction among factors can be tested since there is no crossing of factors. The assumption of random effects when fixed effects are present can lead to problems in analysis and interpretation.

3. Examples of Nested Design

Consider a hypothetical three factor nested design conducted over five days. Each day, three wafers are randomly sampled and three measurements are made on each wafer, one at each of the following sites: top, center, and bottom. The sources of variation are days, wafers, and measurements or sites. To visualize the variation a series of three plots are useful: the average wafer variation from day to day, the variation of sites across the wafers, and a composite plot of variation by sites. See Figures 2,3, and 4 below.

Figure 2. Day to Day Variation

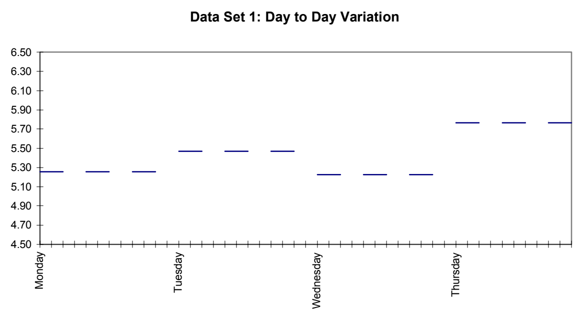


Figure 3. Wafer to Wafer Variation

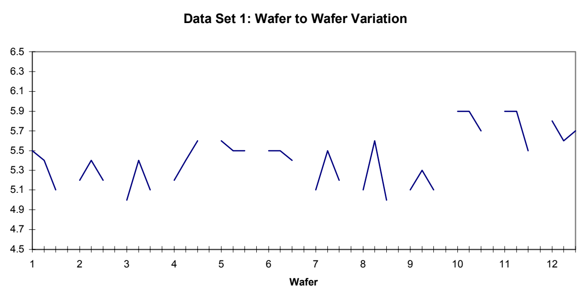
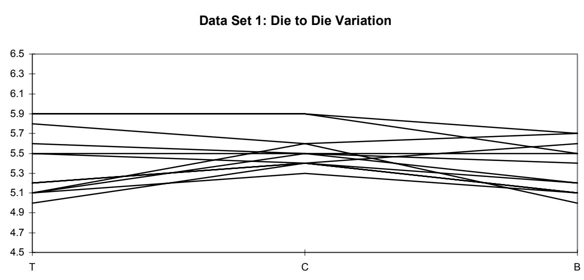


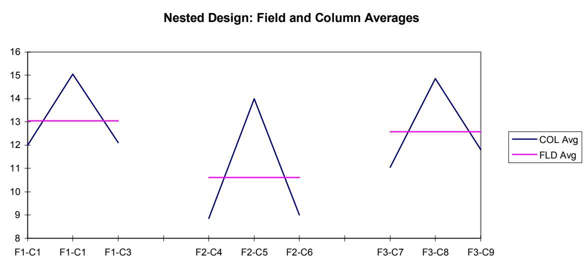
Figure 4. Die to Die Variation



This sequence of graphs shows significant day to day variation, but small and apparently random variation of sites within wafers. There is no systematic pattern associated with fixed effects. Not surprisingly, an analysis of variance revealed that day to day variation accounted for 66% of total variation. There was a small, negative component of variance for wafer to wafer variation, and a 34% contribution from location (die to die variation) within wafer. In this situation, treating wafer to wafer variation as random and negligible appears reasonable.

Consider now another experiment involving three factors: three fields, three columns, and two measurements. The visual display is shown in Figure 5. The horizontal lines represent the average field values across all three column. We note the significant fixed effects associated with the columns within each field: the first column and third columns have low responses compared to the second. There is little measurement variation.

Figure 5. Experiment with Fixed Effects



A components of variance analysis assuming a random effects model produced the following table.

Table 2. ANOVA of Design

ANOVA (Random Effects Model)

		Field to Field	Col to Col	Test to Test
SS	2625.7	19.9	62.6	1.18
DF	1	2	6	9
MS		9.96	10.43	0.13
F Ratio		0.95	79.55	
Sig		43.7%	0.0%	
Comp Var		-0.079	5.149	0.131
% Cont		-1.5%	99.0%	2.5%

We see that the field component is negative, and the column accounts for 99% of the variation. Yet, it is obvious that there is significant field variation as evidenced by the distribution of wafer averages (horizontal lines). It would not be appropriate to neglect this contribution to variance. The problem is an incorrect analysis assuming random effects. In fact, one possible analysis is to treat the factors as non-nested variables and do regression using a polynomial quadratic model with field and columns as fixed factors. The regression output is shown in the table below.

Figure 6 Display of Regression Output

Regression Output
SUMMARY OUTPUT

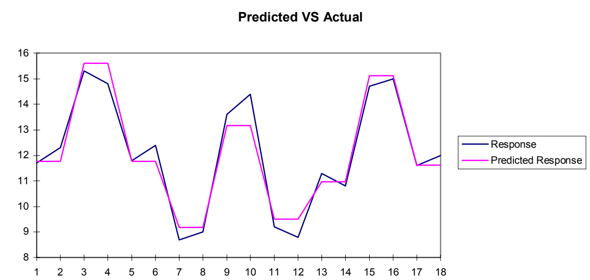
Regression Statistics	
Multiple R	0.9731
R Square	0.9470
Adjusted R S	0.9249
Standard Er	0.6079
Observation	18

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	5	79.24	15.85	42.89	3.03E-07
Residual	12	4.43	0.37		
Total	17	83.67			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	13.172	0.320	41.114	2.78E-14	12.474	13.870
Field	-0.242	0.175	-1.377	0.194	-0.624	0.141
Col	0.167	0.175	0.950	0.361	-0.216	0.549
Field^2	2.192	0.304	7.211	1.07E-05	1.529	2.854
Col^2	-3.833	0.304	-12.612	2.77E-08	-4.496	-3.171
FxC	0.162	0.215	0.756	0.464	-0.306	0.631

We see that only the field and column squares terms are significant. The model fit to the data produced the figure below. We see the excellent fit.



3. Results

The engineer redid the analysis using regression and treating the fixed effects properly. He obtained a model that explained the sources of variation, leading to process improvement.

4. Summary and Conclusions

In variation studies, one must be careful to distinguish fixed and random factors. It is always useful to plot the data. We should consider the possibility of non-nested factors for nested factors and do regression modeling using fixed factors. Analysts must beware of arbitrary applications of components of variance methods, especially when negative components show up. Finally, it is important to understand the error structure of experimental designs.

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