An EXCEL Add-In for Comparing Two Exponential Distributions

David C. Trindade, Sun Microsystems, Inc.

David Meade, AMD, Inc.

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Reliability Analysis Problem

- Exponentially distributed times to failure
- Want to compare two
  - MTTFs (non-repairable components)

  or
  - MTBFs (repairable systems)
- Typically censored data (Time or Count censoring)

Statement of Problem

- Two exponentially distributed data sets of count censored data
- Let T_i be the total time on test, i = 1,2

$$T_i = \sum_{j=1}^{r_i} t_{i(j)} + (n_i - r_i) t_{i(r_i)}$$

- The mean time to failure (MTTF) estimate is

$$\hat{\theta}_i = T_i / r_i$$

Distributions of Statistics

- T_i has a distribution given by

$$2T_i / \theta_i \sim \chi^2_{(2r_i)}$$

- The ratio

$$\frac{2T_1 / 2r_1\theta_1}{2T_2 / 2r_2\theta_2} = \frac{\hat{\theta}_1\theta_2}{\hat{\theta}_2\theta_1}$$

has an *F*-distribution with $(2r_1, 2r_2)$ degrees of freedom

Procedure

- Set up two-sided 100γ% confidence interval for the ratio θ_1 / θ_2
- Confidence limits are

$$\underset{\sim}{\rho} = \left(\hat{\theta}_1 / \hat{\theta}_2\right) / F\left[(1+\gamma)/2; 2r_1; 2r_2\right]$$

$$\tilde{\rho} = \left(\hat{\theta}_1 / \hat{\theta}_2\right) \cdot F\left[(1+\gamma)/2; 2r_2; 2r_1\right]$$

- If interval **excludes** 1, then θ_1 and θ_2 differ statistically at the specified significance level

Example (Count Censored)

Two exponentially distributed data sets

$\hat{\theta}_1 = 12{,}600$ $\hat{\theta}_2 = 8{,}800$

$r_1 = 16$ $r_2 = 22$

Ratio $(\hat{\theta}_1 / \hat{\theta}_2)$ 1.43

95% CI: LCL 0.76 UCL 2.80

Interval includes 1.

Not significantly different.

Time Censored Data

The ratio

$$\frac{(1+0.5/r_1)}{(1+0.5/r_2)}\frac{\theta_2\hat{\theta}_1}{\theta_1\hat{\theta}_2}$$

has an *F*-distribution with $(2r_1+1, 2r_2+1)$ degrees of freedom.

Procedure

- Set up two-sided 100γ% confidence interval for the ratio θ_1 / θ_2
- Confidence limits are

$$\underset{\sim}{\rho} = \frac{(\hat{\theta}_1 / \hat{\theta}_2)(1 + 0.5 / r_1)}{(1 + 0.5 / r_2)} / F\left[(1+\gamma)/2; 2r_1 + 1; 2r_2 + 1\right]$$

$$\tilde{\rho} = \frac{\left(\hat{\theta}_1 / \hat{\theta}_2\right)(1 + 0.5 / r_1)}{(1 + 0.5 / r_2)} \cdot F\left[(1+\gamma)/2; 2r_2 + 1; 2r_1 + 1\right]$$

- If interval **excludes** 1, then θ_1 and θ_2 differ statistically at the specified significance level

Example (Time Censored)

Two exponentially distributed data sets

$\hat{\theta}_1 = 120$ $\quad\quad$ $\hat{\theta}_2 = 55$

$r_1 = 10$ $\quad\quad$ $r_2 = 15$

Ratio $(\hat{\theta}_1 / \hat{\theta}_2)$ 2.18

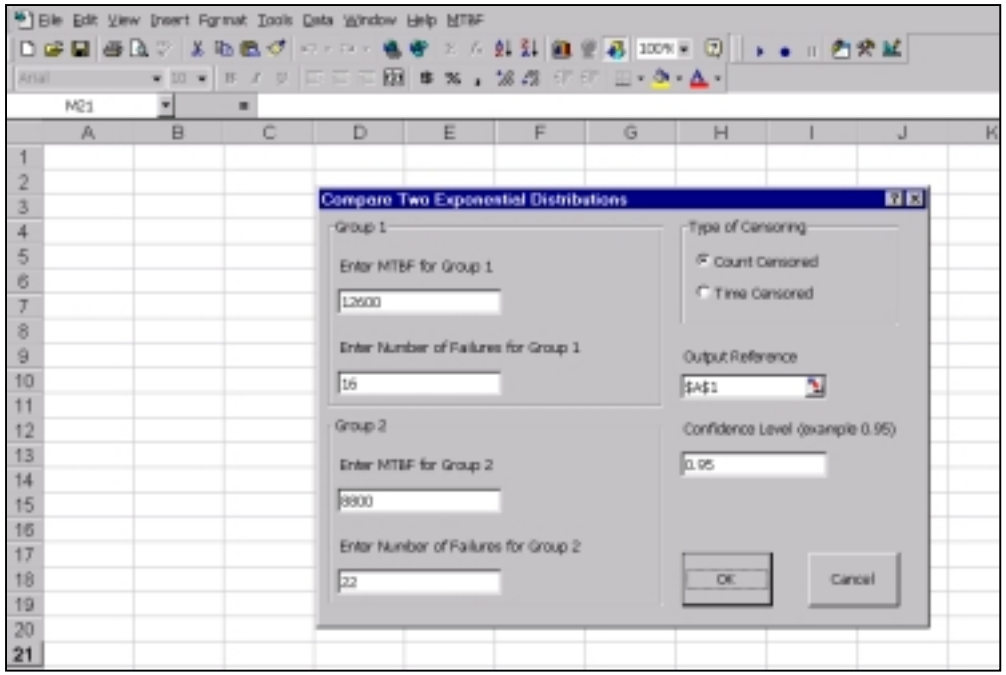
95% CI: LCL 1.03 $\quad\quad$ UCL 5.10

Interval excludes 1.

Significantly different.

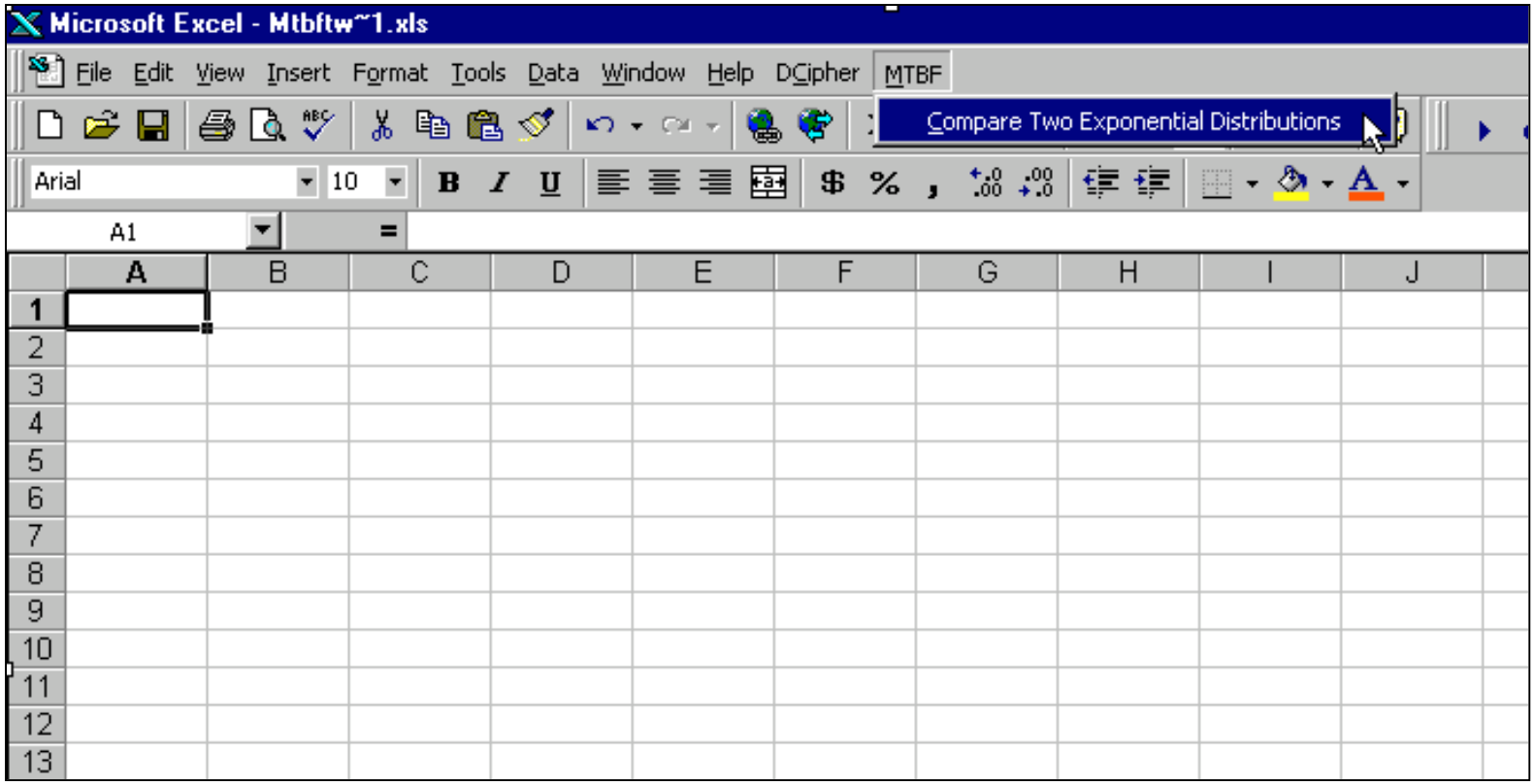
EXCEL Add-In

- This visual basic add-in makes the comparison of exponential distributions a snap!



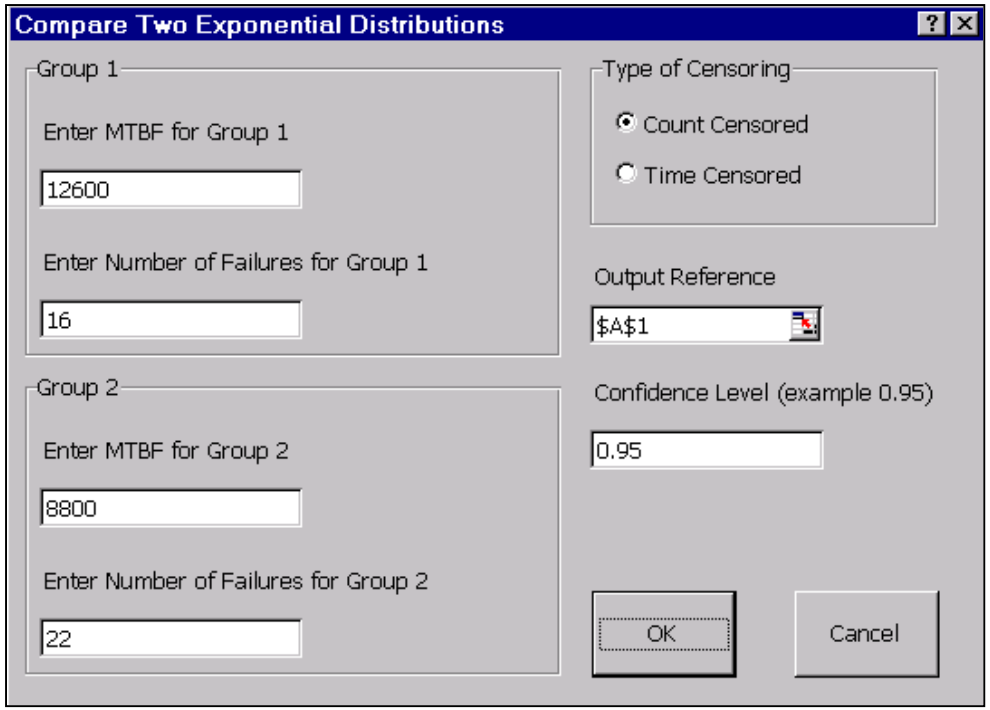
Software Example

Simply Execute Our User-friendly Macro By Using the Pull Down Menu Provided.



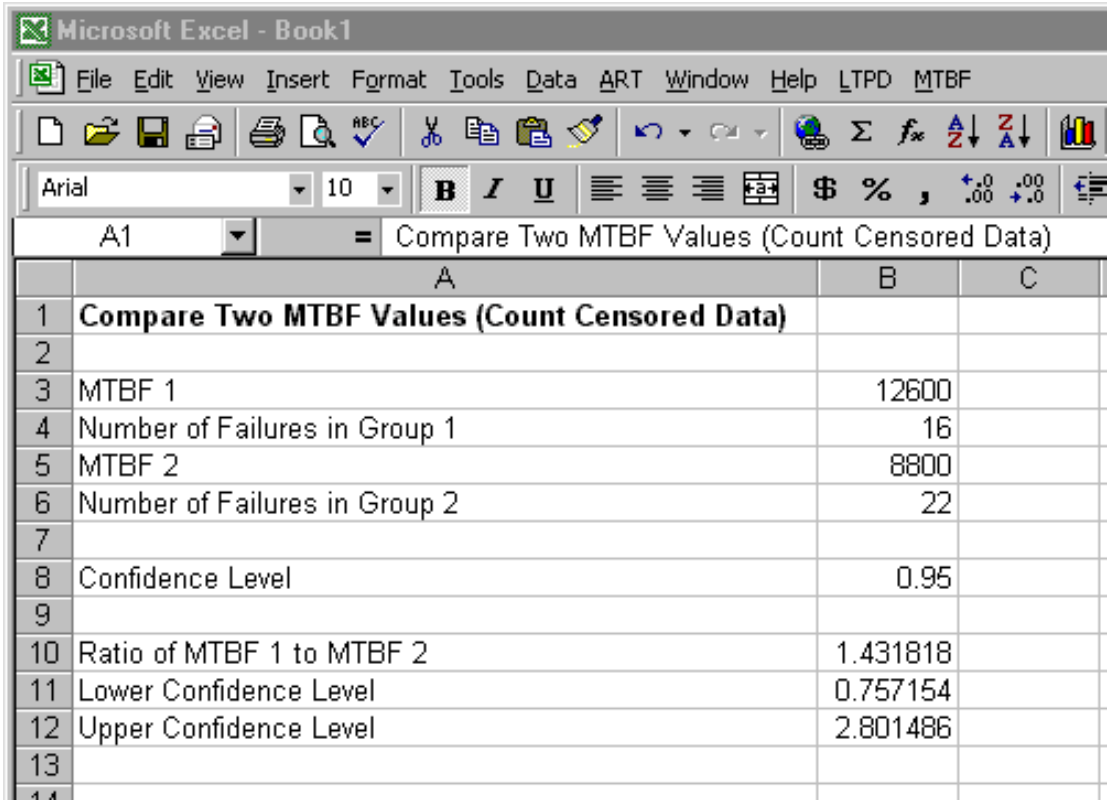
Software Example Cont.

- **Select the Censoring Type.**
- **Fill In The MTBF and Number of Failures for Each Distribution, as shown below.**



Software Example Cont.

Click the "OK" Button, and Let the Software Do the Rest.
We Do All of the Calculations For You!



	A	B	C
1	**Compare Two MTBF Values (Count Censored Data)**		
2			
3	MTBF 1	12600	
4	Number of Failures in Group 1	16	
5	MTBF 2	8800	
6	Number of Failures in Group 2	22	
7			
8	Confidence Level	0.95	
9			
10	Ratio of MTBF 1 to MTBF 2	1.431818	
11	Lower Confidence Level	0.757154	
12	Upper Confidence Level	2.801486	
13			

Summary

- Simple procedures provided to compare two exponential distributions
- Excel add-in availability at

  www.trindade.com/mttf.htm

References

- Bain, L.J. and Englehardt, M. (1991) *Statistical Analysis of Reliability and Life-Testing Models*, 2nd Ed., Marcel Dekker, New York
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- Nelson, W. (1982) *Applied Life Data Analysis*, John Wiley and Sons, New York
- Lawless, J.F. (1982) Statistical Models & Methods for Lifetime Data, John Wiley and Sons, New York