CONFIRMING TRENDS IN REPAIRABLE SYSTEM RELIABILITY

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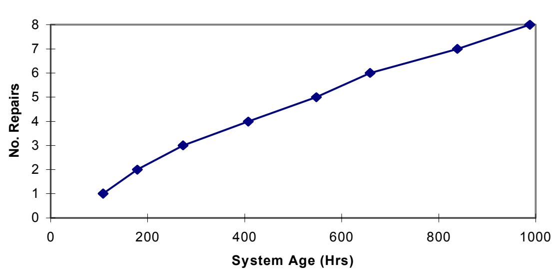
The times between equipment failure in a renewal process are independent and identically distributed, that is, the times arise from a single population. If the population distribution is exponential, the result is a homogeneous Poisson process (HPP). However, a renewal process is not valid when the equipment reliability degrades or improves. We discuss some graphical and analytical tools for detecting trends in repairable systems. In particular, the reverse arrangement test (RAT) is a simple but powerful statistical procedure to check for trends and randomness and thereby verify the validity of an assumed renewal process. We provide an example application of the methodology to a case study in the semiconductor industry.

Introduction

The basic problem involves **i**ncorrect analysis of repairable system reliability data in which the application of techniques for the analysis of nonrepairable component data can lead to misleading conclusions. To illustrate, we consider a case study on a repairable system. The repair history showed failures at system ages 108, 178, 273, 408, 548, 658, 838, and 988 hours. The time to make the repair at each failure point is ignored. The cumulative repair plot is shown below.

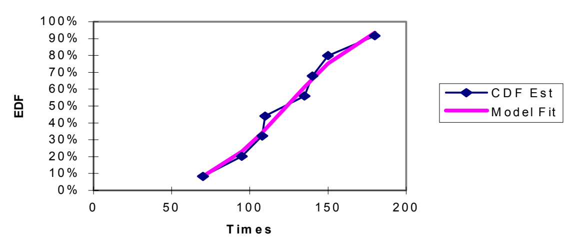
Cumulative Repairs vs System Age



The engineers analyzed the repairable system data in a traditional manner (See Tobias and Trindade.) by taking times-between-repairs and treating them as a group of independent and identically distributed observations arising from a single population of failure times. Methods for the analysis of nonrepairable components were used including Weibull probability plotting of data, parameter estimation, and model fitting. Thus, the times between repairs (called the interarrival times), that is, 108, 70, 95, 135, 140, 110, 180, and 150 hours, were sorted and plotted on Weibull probability paper.

Since the plot showed reasonable fit to a straight line, the parameters of the Weibull distribution were estimated and used for assessing system performance. The shape parameter estimate was m = 3.65 and the scale parameter, c, estimate was c = 137 hours. The graph below shows the Weibull model fit to the empirical distribution function (EDF) obtained by treating the data as a single population of failure times.

Weibull Model Fit to EDF



Because of the reasonable fit using the Weibull model, the engineers concluded that the repair times followed a Weibull distribution. Also, the interpretation followed that the "failure rate" is increasing because the shape parameter, m, is greater than 1.0. Consequently, the equipment engineers thought the machine needed to be brought down for repair and maintenance. We shall return to this case study later in this paper, but first, we develop some useful concepts.

Repairable Systems Concepts

A system is repairable if it can be restored to satisfactory operation by any action, including replacement of components, changes to adjustable settings, swapping of parts, or even a sharp blow with a hammer. Examples include TV's, automobiles, and production equipment.

There are important reliability considerations for repairable systems. The failures occur sequentially in time. The times between failures may not be independent and identically distributed (i.i.d.) observations from a single population, that is, a renewal process. There may be stability, improvement, or degradation in the rate of repairs. The order in which failures occur is important. In contrast, for nonrepairable component analysis, the order of failures is ignored and times between failures are considered independent observations from a single population

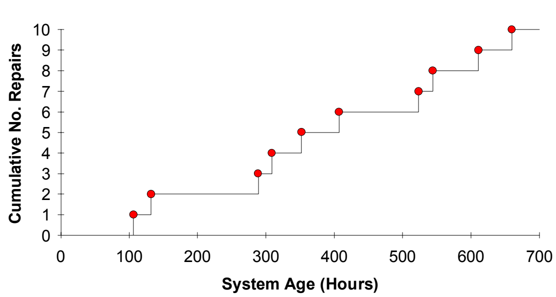
For repairable systems, there may be trends, indicating improvement or deterioration, which affect maintenance schedules, spare parts provisions, warranty costs, reliability growth objectives, etc. Analysis may apply to either a single system, to understand behavior for possible reliability improvement of existing or future systems, or to many copies of the systems, to estimate the repair rate of a population of system or to specify burn-in effectiveness
In this paper, we will concern ourselves with analysis of the single system.

The times to repair are a function of many factors, including basic system design, operating conditions, type of repairs, quality of repairs, materials used, etc. For a single component system, restoration to "like new," such as replacement of the failed component with one from same population, implies a renewal process (i.i.d.). However, even replacement with identical components is no guarantee of a renewal process! (See Usher.) If inter-repair times are not i.i.d., renewal model is not valid and special techniques for analysis are required. In a renewal process, the times between failures are i.i.d. from a single population. There is no trend, that is, the repair rate is stable. Reliability analysis methods for non-repairable components have applicability.

Analysis of Renewal Process

Consider a single system for which the times to make repairs are ignored. Ten failures are reported at the system ages (in hours): 106, 132, 289, 309, 352, 407, 523, 544, 611, 660. The most common data graph is called the cumulative plot: the cumulative number of repairs is plotted against the system age. For the data shown, the cumulative plot is:

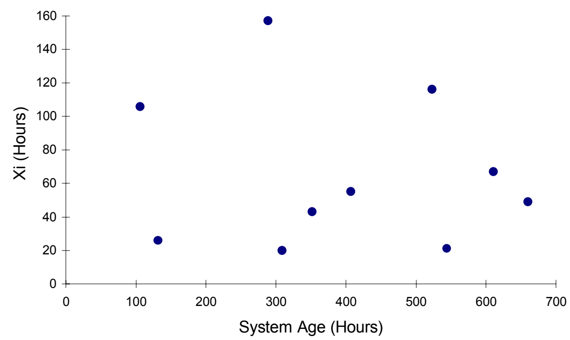
Figure 10.1 Cumulative Plot



Under a renewal process, the times between failures are i.i.d., that is, from a single population having a fixed mean (average) repair time. Consequently, the cumulative plot should appear to follow a straight line.

Analysis of Interarrival Times

Look at the times between repairs, called the interarrival times: 106, 26, 157, 20, 43, 55, 116, 21, 67, 49 hours. A useful chart is a plot of the interarrival times versus the system age at repair.



Interarrival Times Versus System Age

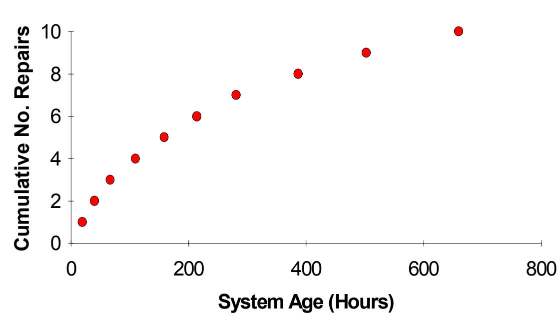
If a renewal process exists, we can treat the sample of ten (assumed) independent observations (that is, the interarrival times) as arising from a single population. We can analyze the data using methods for non-repairable components. Thus, we can sort the data and plot on probability paper or use MLE methods.

Suppose the interarrival times X_i are i.i.d. with exponential probability ddensity function (pdf) having failure rate λ. A renewal process in which the interarrival distribution is exponential is called a homogeneous Poisson process (HPP). The expected value for N(t) is λt. The mean time to the *k*th repair is *k* / λ.

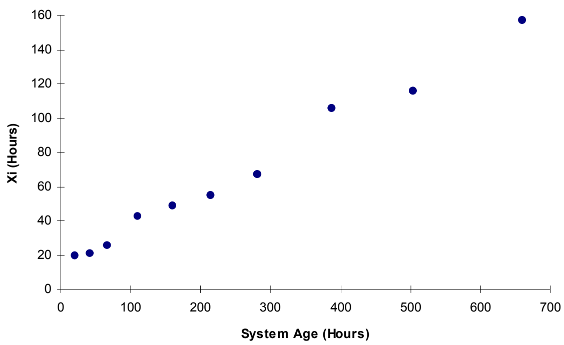
Graphical Analysis of Non-Renewal Processes

Suppose the observed consecutive repairs times were 20, 41, 67, 110, 159, 214, 281, 503, 660 hours. The cumulative plot is shown below. The curvature suggests a decreasing frequency of repairs, that is, an improving failure rate.

Figure 10.4 Cumulative Plot - Improving



The interarrival times are 20, 21, 26, 43, 49, 55, 67, 106, 116, 157. These are exactly the same interarrival times as those for the renewal process! The plot of interarrival times versus system age is shown next.



Now the order in which the interarrival times appear is important. Again we confirm an improving trend. We cannot use standard non-repairable methods, such a probability plotting, to analyze such data by assuming the time between repairs are independent observations from a single population.

Testing for Trends and Randomness

Model assumptions should be verified. We recommend a sequence of checks: Plot the data. Check for trend. If confirmed, check for NHPP or other nonstationary models. If no trend, check for identically distributed and independent. If i.i.d., we have a renewal process. If i.d. and not independent, check for other model. If not i.i.d., possibly subdivide data. If renewal, check if exponential distribution for interarrival times holds, giving a HPP. If not, another models applies or use distribution free methods.

Analytical Tools to Check Trend

Laplace Test is used to deterrmine whether or not an observed series of events is a HPP. The test statistic is

$$L = \frac{\sum_{i=1}^{n} T_i - \frac{nT}{2}}{T\sqrt{n/12}}$$

Reverse Arrangement Test (RAT)

This is a nonparametric test (no distribution assumed). Consider set of interarrival times occurring in the sequence

$$X_1,\ X_2,\ \ldots,\ X_n$$

Define a reversal as each instance of an earlier repair being smaller than subsequent times, that is,

$$X_i < X_j$$

for $i < j$, where $i = 1,\ldots,n-1$ and $j = 2,\ldots,n$

As an example of counting reversals, consider a system with repairs at ages: 25, 175, 250, and 350 hours. The interarrival times are 25, 150, 75, 100 hours. There are 3 reversals for the first time of repair 25, since that time is less than next three. There are zero reversals for the second time 150 which is larger than the next two. There is one reversal for the time 75 which is smaller than the last. Thus, the series of interarrival times has 3+0+1=4 reversals.

The RAT criteria are: too many reversals indicate increasing trend; too few, consistent with decreasing trend. Statistically, we can calculate, for *n* repair times, tables of critical values for a specific number of reversals (see Tobias and Trindade) to reject evidence of no trend in the data series and thereby conclude a trend does exist.

Determining RAT Critical Values

Consider n = 4 observations, designated

$$X_1, X_2, X_3, X_4$$

There are 4!=24 possible permutations. We can show that the maximum number of reversals for a series of *n* times is $n(n-1)/2$. So for $n = 4$, we have a maximum of 4(3)/2 = 6. By counting the number of reversals for each permutation, we can calculate the probability of zero to 6 reversals occurring by chance. For our n = 4 example, the sequence

$$X_1 < X_2 < X_3 < X_4$$

is the only permutation with six reversals. There are only 3 permutations that give 1 reversal

$X_4X_3X_1X_2$, $X_3X_4X_2X_1$, and $X_4X_2X_3X_1$

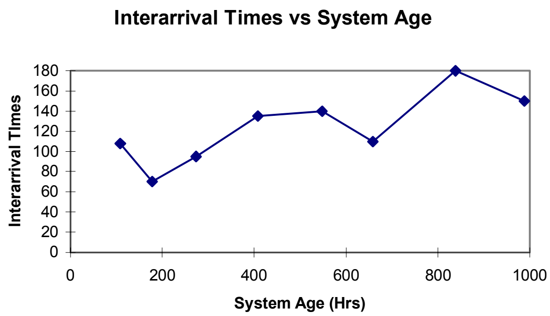
and so the probability of exactly 1 reversal is 3/24. Next, we can determine which permutations give 2 reversals and so on.

For $n = 4$, the probability of 0, 1, 2, 3, 4, 5, 6 reversals is 1/24, 3/24, 5/24, 6/24, 5/24, 3/24, 1/24, respectively. Since 1/24 = 4.2%, we see that 0 or 6 reversals is significant at the upper or lower 5% significance level.

We can create tables of critical reversal numbers for different *n*. See Tobias and Trindade.

Case Study Example

Recall the system experienced repairs at the following ages 108, 178, 273, 408, 548, 658, 838, 988. Is there any evidence of a trend? First, we check graphically. The interarrival times versus system age plot is shown below.



There appears to be a trend visible in the plot. How significant is it? We need a statistical approach that tells us the likelihood of such a pattern if there really is no trend. What is the probability of the observed sequence of repair times occurring by chance alone under a renewal process? We apply RATS to the interarrival times, which are 108, 70, 95, 135, 140,110, 180, 150. There are 5+6+5+3+2+2+0=23 reversals Comparison to the critical table shows 22 reversals in 8 items is significant at the 5% level. Hence, we reject any renewal process, and in particular, the HPP, as a suitable model. With at least 95% confidence, we state that the system is improving in time.

Results/Implementation

The correct analysis showed an improving trend in the repairable system history. Incorrect analysis lead to the belief that maintenance was necessary to restore reliability when such action might have made the reliability worse. By using correct procedures to detect the trend, the realization of the improvement was made and corrective action halted. Search for the source of the improvement was instead addressed leading to adoption of new techniques for repair. The result was improved reliability for the existing system and the prospect of improved reliability for future systems

By not performing unnecessary maintenance considerable savings in money and cycle time was possible. Unnecessary repairs could have made the reliability worse. If correct techniques are not employed, reliability improvement could be missed. Engineers concluded that RAT is a simple test to apply., and graphical procedure are effective.

Summary

We have discussed various processes for repairable systems for both renewal and non-renewal situations. We have presented both graphical and analytical methods for revealing trends. We have reviewed a simple procedure called RAT for performing a nonparametric test for trend in repairable system data. Important to verify assumptions in reliability analysis of repairable systems. Analysis of repairable systems with techniques for non-repairable components can be misleading and costly. Powerful graphical and analytical techniques exist for detecting trends in repairable system reliability.

References

For further discussion of techniques for the analysis of data from repairable systems, see the text, *Applied Reliability*, 2nd edition, by Paul Tobias and Dave Trindade, published in 1995 by Van Nostrand Reinhold, New York, NY.

See also the paper by John Usher entitled “Case Study: Reliability Models and Misconceptions,” in *Quality Engineering*, 6(2), pages 261-271 (1993-1994).

For an application of RAT, see the article by R.F. De Le Mare entitled “Testing for Reliability Improvement or Deterioration in Repairable Systems,” in *Quality and Reliability Engineering International*, Vol. 8, pages 123-132 (1991).

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