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Multivariate Process Control with Radar Charts

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Introduction

- Multivariate statistical process control
- Monitoring with separate $\bar{X}$ control charts
- Hotelling T^2 statistic
- Radar or web plots
- Microsoft EXCEL capabilities
  - Matrix manipulation
  - Graphical

Outline of Presentation

- Example of process with two quality characteristics per sample
- Univariate and multivariate considerations
- Calculations and graphical analysis in EXCEL
- Implications

Vocabulary

- Multivariate
- Matrix representation
- Hotelling T^2 statistic
- Radar or web plots
- Subgroups Vs. individual observations

Multivariate SPC

- Consider a process in which two quality characteristics ($p = 2$) are measured on each of four samples ($n = 4$) within a subgroup.
- Data on twenty subgroups are available ($k = 20$)
- Data is from Thomas P. Ryan, *Statistical Methods for Quality Improvement*

Sample Data for Each Subgroup

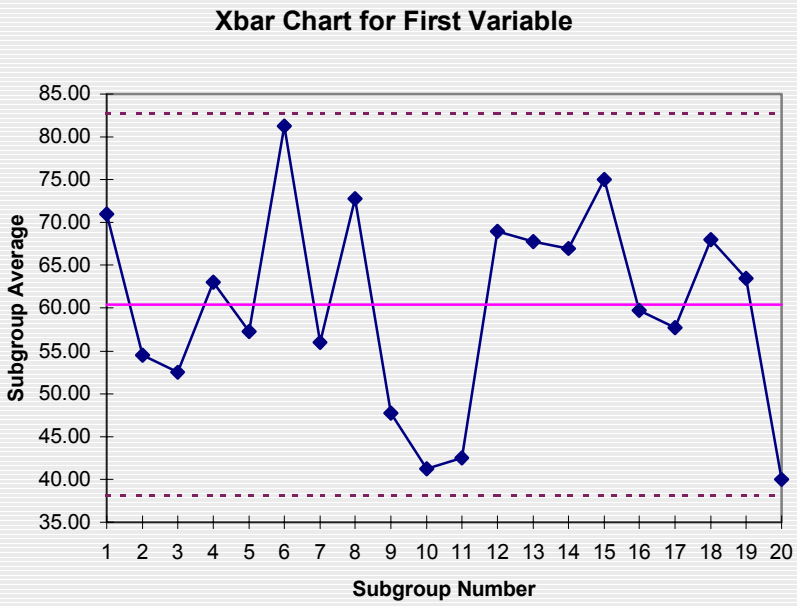
Subgroup Number	First Variable X				Second Variable X			
1	72	84	79	49	23	30	28	10
2	56	87	33	42	14	31	8	9
3	55	73	22	60	13	22	6	16
4	44	80	54	74	9	28	15	25
5	97	26	48	58	36	10	14	15
6	83	89	91	62	30	35	36	18
7	47	66	53	58	12	18	14	16
8	88	50	84	69	31	11	30	19
9	57	47	41	46	14	10	8	10
10	26	39	52	48	7	11	35	30
11	46	27	63	34	10	8	19	9
12	49	62	78	87	11	20	27	31
13	71	63	82	55	22	16	31	15
14	71	58	69	70	21	19	17	20
15	67	69	70	94	18	19	18	35
16	55	63	72	49	15	16	20	12
17	49	51	55	76	13	14	16	26
18	72	80	61	59	22	28	18	17
19	61	74	62	57	19	20	16	14
20	35	38	41	46	10	11	13	16

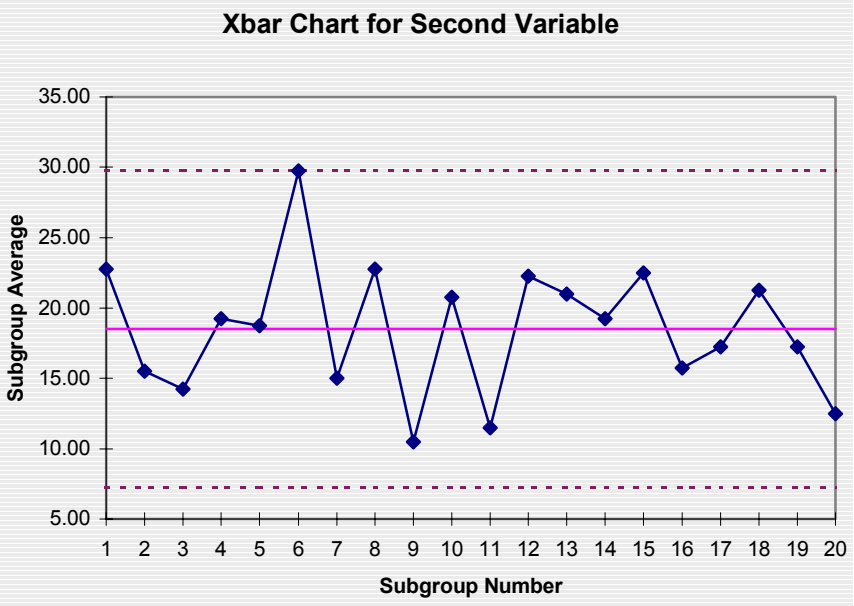
Multivariate observations are (72,23), (84,30), …, (46, 16).

Subgroup Averages

Subgroup Number	Subgroup Averages	
	First Variable	Second Variable
1	71.00	22.75
2	54.50	15.50
3	52.50	14.25
4	63.00	19.25
5	57.25	18.75
6	81.25	29.75
7	56.00	15.00
8	72.75	22.75
9	47.75	10.50
10	41.25	20.75
11	42.50	11.50
12	69.00	22.25
13	67.75	21.00
14	67.00	19.25
15	75.00	22.50
16	59.75	15.75
17	57.75	17.25
18	68.00	21.25
19	63.50	17.25
20	40.00	12.50

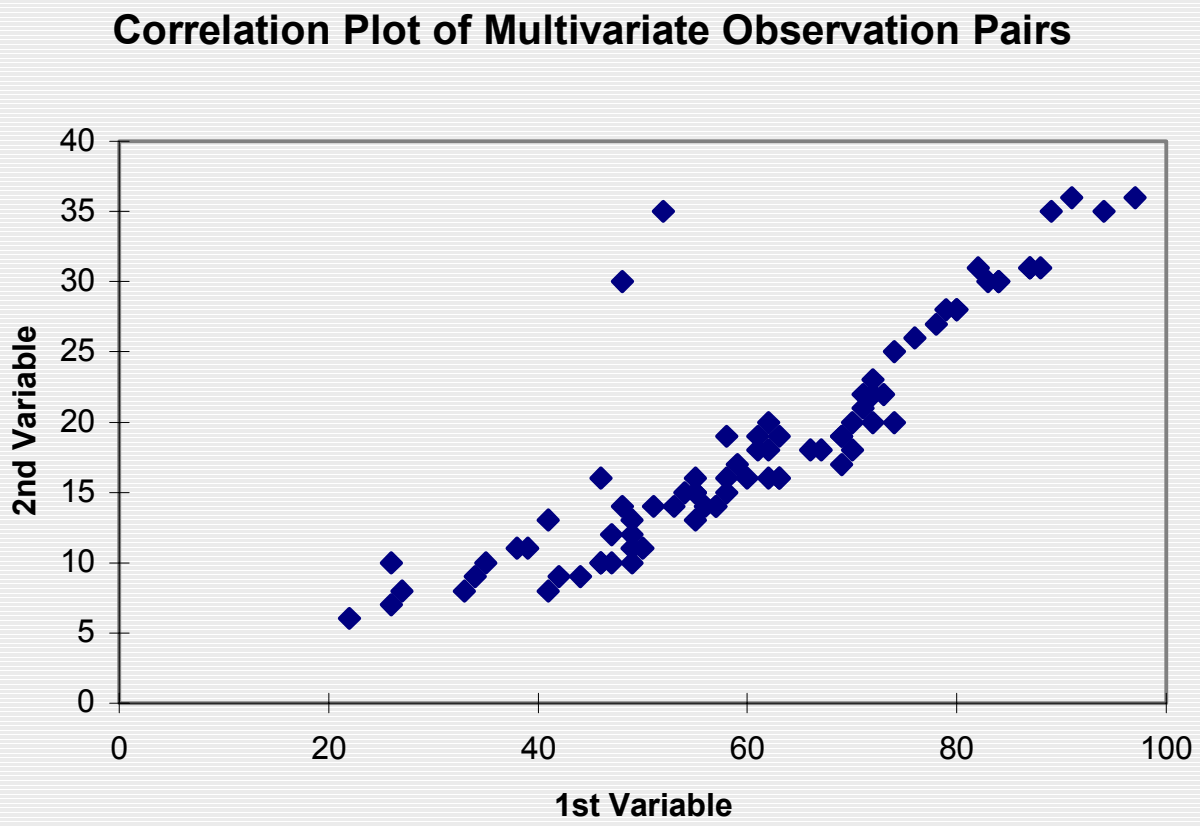
Xbar Chart for Each Variable





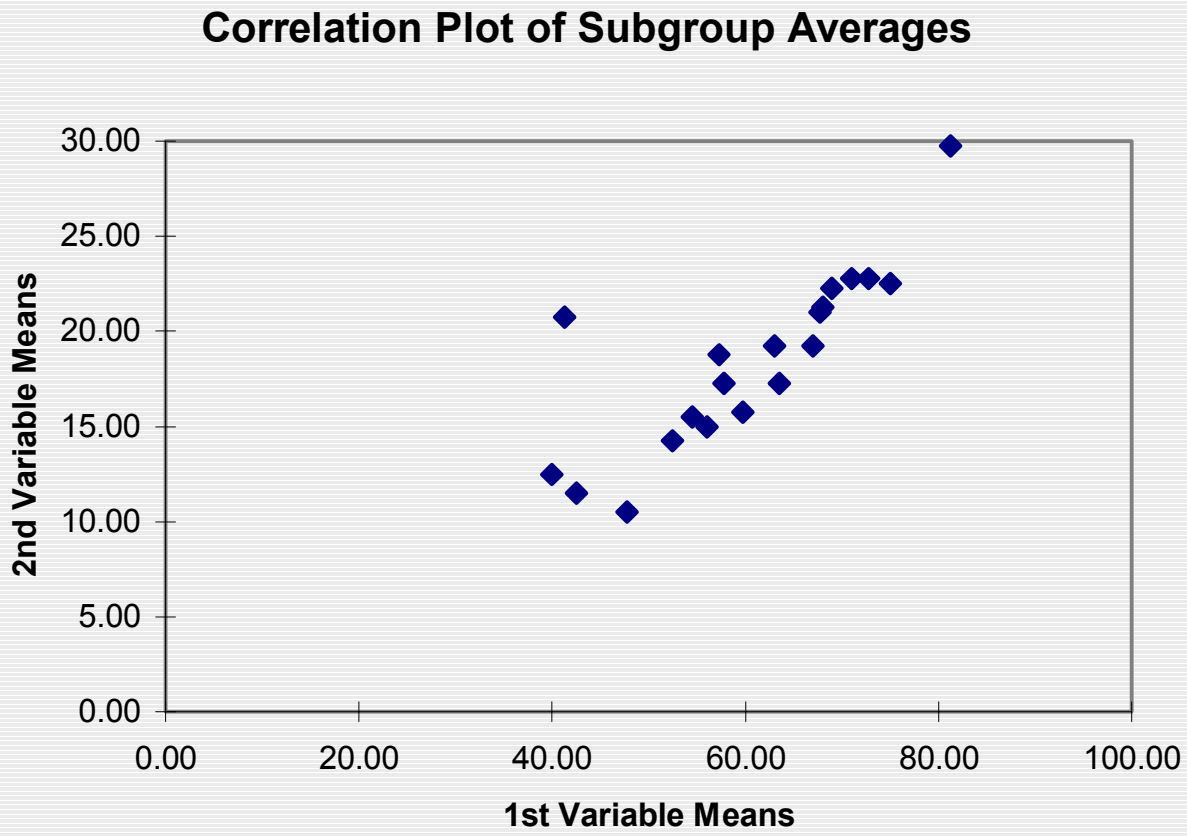
Subgroup 6 produces a almost out-of-control situation for the first and second variables.

Variables are Correlated



Two points stand out.

Correlation of Subgroup Averages



One point stands out.

Time Order Lost in Correlation Plots

- Need a way to plot correlation behavior in time.
- Employ Hotelling's T^2 statistics along with control limits.

Calculating Hotelling's T^2

Start with sample variances and covariances

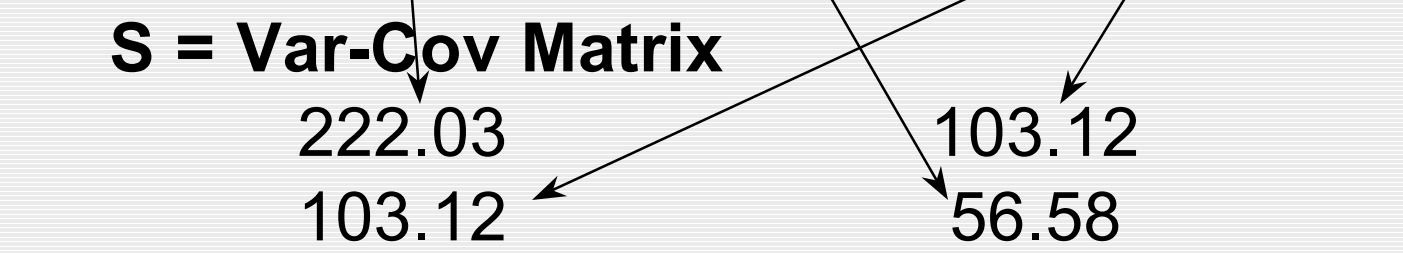
Subgroup Variance		Covariance
First Variable	Second Variable	Between Variables
239.33	80.92	139.00
559.00	113.67	248.00
471.00	44.25	140.17
284.00	77.58	148.33
880.92	136.92	333.42
176.25	68.25	109.42
64.67	6.67	20.67
296.92	90.92	162.92
44.92	6.33	16.83
132.92	190.92	149.08
248.33	25.67	74.67
284.67	76.92	147.00
132.92	54.00	82.00
36.67	2.92	2.33
162.00	69.67	105.67
99.58	10.92	32.25
154.25	35.58	74.08
96.67	24.92	48.33
53.67	7.58	15.83
22.00	7.00	12.33

Caution: EXCEL covariance function divides by *n* instead of *n*-1. Need to adjust for sample covariance.

Variance-Covariance Matrix

Use average of sample variances and covariance across subgroups to form variance-covariance matrix.

	Variance First Variable	Variance Second Variable	Covariance Between Variables
Averages	222.03	56.58	103.12



Inverse of Var-Covar Matrix

$\mathbf{S^{-1}}$

0.0293	-0.0534
-0.0534	0.1151

In EXCEL, highlight area for inverse.

Then type =minverse(*cell array containing var-covar matrix* ***S***).

Press CNTRL-SHIFT-ENTER.

Deviations of Subgroup Means from Overall Means

Create matrix:

$$\left(\overline{X} - \overline{\overline{X}}\right)$$

Deviations of Subgroup Means from Overall Means	
First Variable	Second Variable
10.63	4.26
-5.88	-2.99
-7.88	-4.24
2.63	0.76
-3.13	0.26
20.88	11.26
-4.38	-3.49
12.38	4.26
-12.63	-7.99
-19.13	2.26
-17.88	-6.99
8.63	3.76
7.38	2.51
6.63	0.76
14.63	4.01
-0.63	-2.74
-2.63	-1.24
7.63	2.76
3.13	-1.24
-20.38	-5.99

Transpose Matrix

$$\left(\overline{\boldsymbol{X}} - \overline{\overline{\boldsymbol{X}}}\right)'$$

Transpose Matrix of Subgroup Average Deviations from Overall Means

First Variable	10.625	-5.875	-7.875	2.625	-3.125	20.875	-4.375	12.375	-12.625	-19.125	-17.875	8.625	7.375	6.625	14.625	-0.625
Second Variable	4.2625	-2.9875	-4.2375	0.7625	0.2625	11.2625	-3.4875	4.2625	-7.9875	2.2625	-6.9875	3.7625	2.5125	0.7625	4.0125	-2.7375

In EXCEL, highlight area for transpose.

Then type =transpose(*array cells for matrix to be transpose*).

Press CNTRL-SHIFT-ENTER.

Hotelling T^2 Calculations

For each subgroup, calculate

$$T^2 = n\left(\overline{X}_i - \overline{\overline{X}}\right)S^{-1}\left(\overline{X}_i - \overline{\overline{X}}\right)'$$

In EXCEL, do in two stages: highlight area of size *1xp*.

Then, type =mmult(*1xp cell array containing **i**th subgroup deviations from **i**th overall mean, pxp cell array containing* S^{-1})

Press CNTRL-SHIFT-ENTER.

Highlight single cell.

Then, type =mmult(*1xp cell array containing results of last calculations, px1 transpose array containing **i**th subgroup deviations from **i**th overall mean*)

Press CNTRL-SHIFT-ENTER.

Subgroup T^2

Subgroup T^2
2.24
0.65
1.27
0.22
1.53
8.98
1.32
3.77
4.95
63.76
6.55
1.37
1.36
3.26
7.41
2.76
0.12
1.33
3.50
13.04

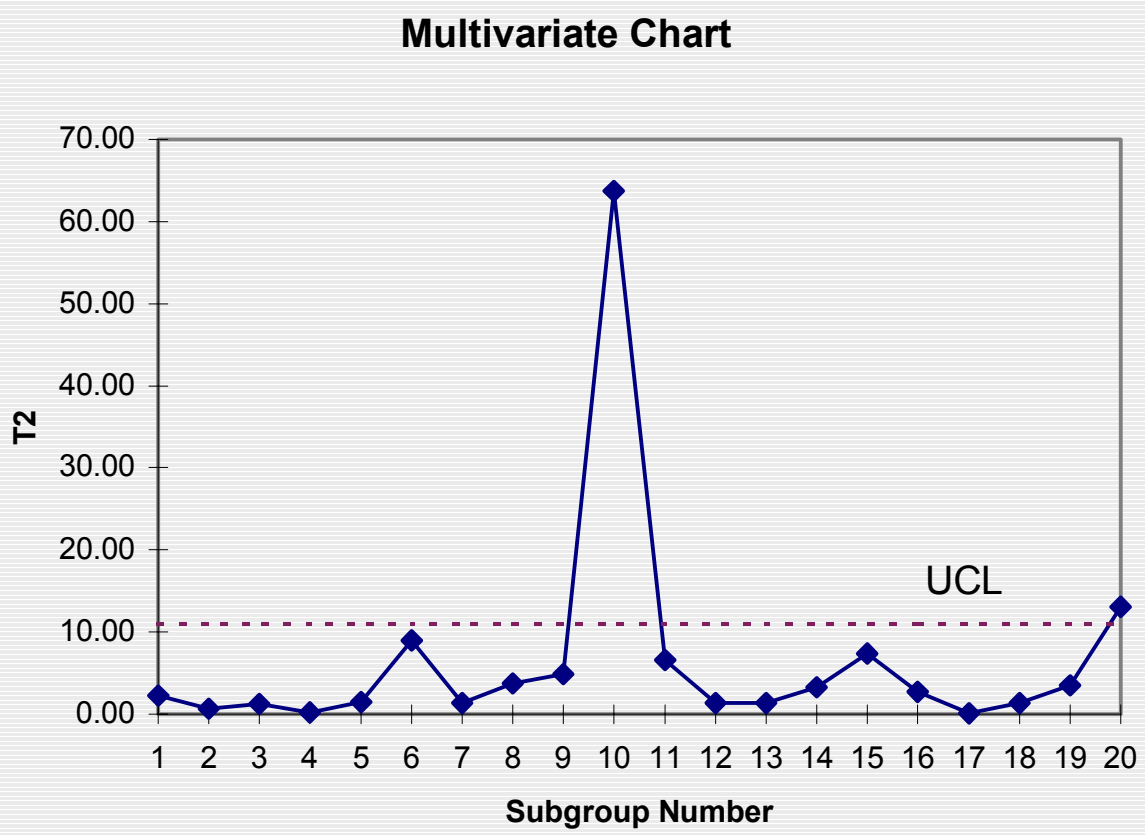
Control Limit for T^2

$$UCL = \left(\frac{knp - kp - np + p}{kn - k - p + 1} \right) F_{\alpha}\left(p, kn - k - p + 1\right)$$

Here, k = 20 subgroups, n = 4 samples per subgroup, and p = 2 quality characteristics measured per sample. Choose alpha = 0.0054.

UCL = 11.04

Control Chart for T^2



Interpretation

The individual Xbar charts did not detect the out-of-control situation at subgroup #10. The **bivariate** process is out-of-control. Something was wrong with the process when the data subgroup #10 were obtained. Also problem with subgroup #20.

We need to remove the out-of-control data points and recalculate limits.

The UCL is used for testing for control of the entire set of subgroups. Monitoring future values requires a different control limit.

Radar Plots

- What do we do when the number of quality characteristics is greater than 2?
- Can create individual control charts for each characteristic and a separate T^2 chart.
- Alternative is to use one chart, summarizing each quality characteristic relative to its control limits and add a ray for T^2.

Radar Plot Coding

- One ray per quality characteristic.
- Each quality characteristic is standardized by subtracting process mean and dividing by the standard deviation.
- Control limits are normally at -3 to +3 for each characteristic
- To plot on radar chart, we add 6 to all coded readings. Thus, control limits are at 3 and 9.

Control Polygons

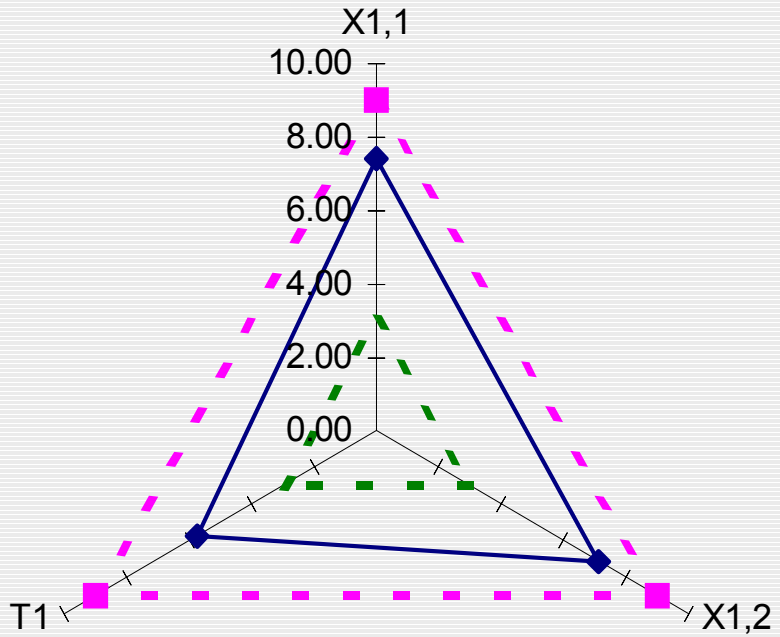
- We get a regular inner polygon, a regular outer polygon.
- The T^2 statistic is scaled so that the critical value is at the outer polygon limit (9) and $T^2 = 0$ is the inner polygon limit (3).

EXCEL Input for Radar Plots

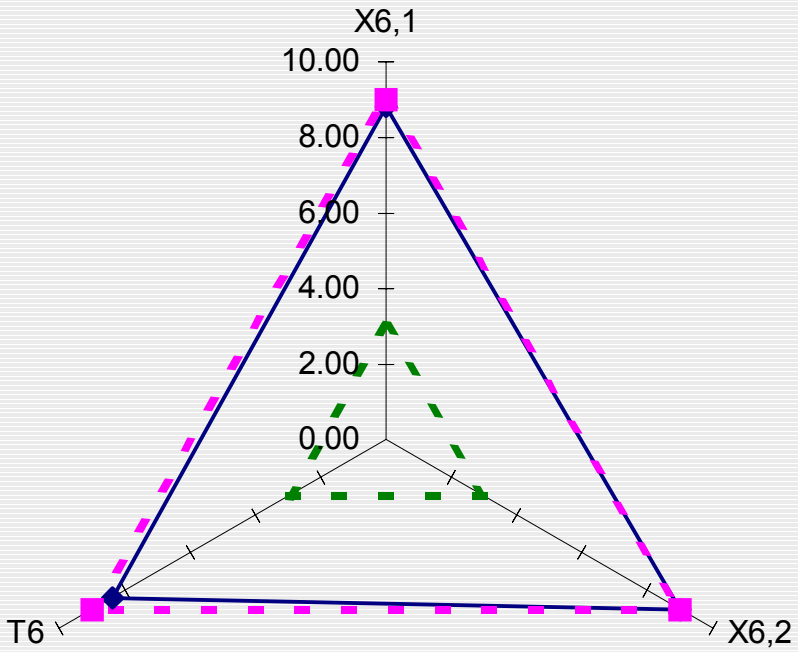
Subgroup #1

	$\bar{X}_{1,1}$	$\bar{X}_{1,2}$	T1
Data	7.43	7.13	5.73
UCL	9	9	9
LCL	3	3	3

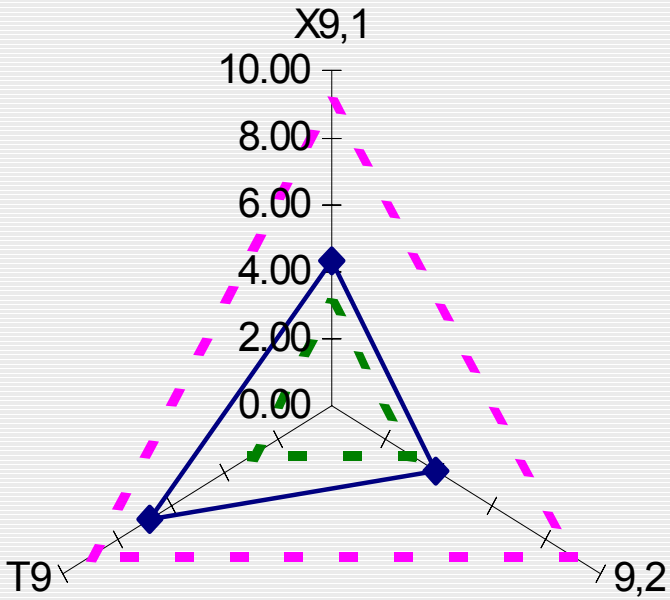
Radar Plot: Subgroup #1



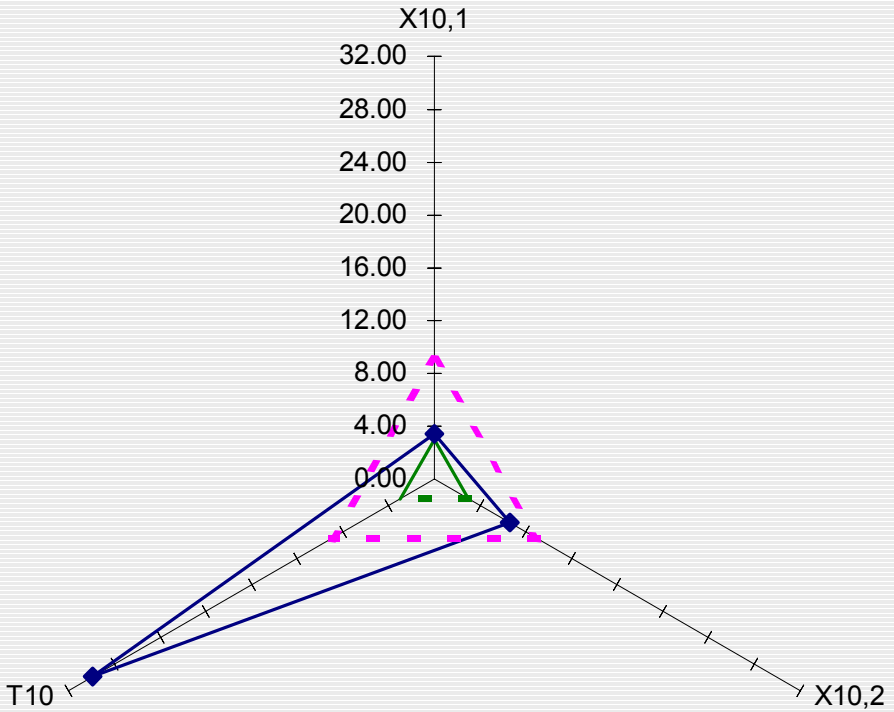
Radar Plot, Subgroup #6



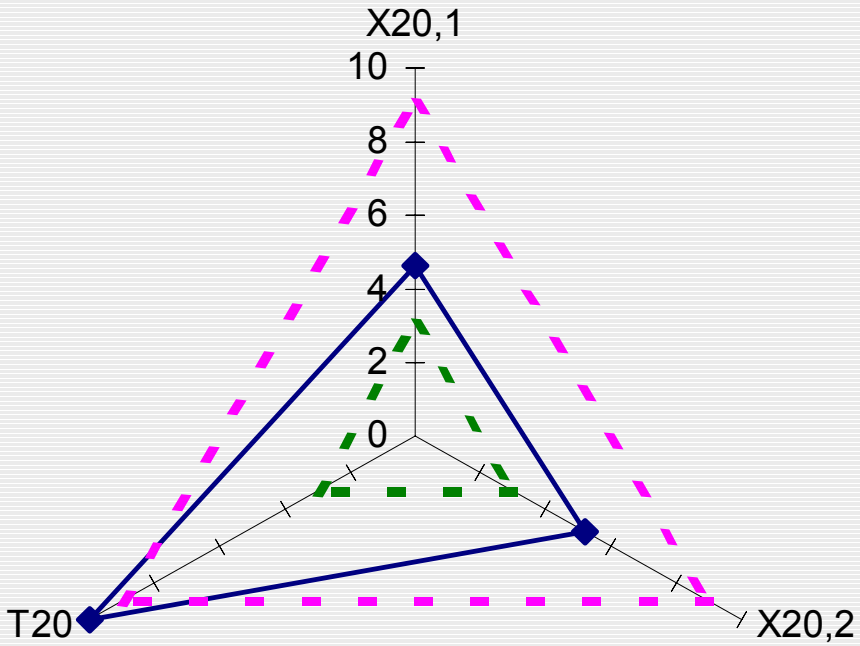
Radar Plot, Subgroup #9



Radar Plot, Subgroup #10



Radar Plot, Subgroup #20



Summary

- Multivariate analysis
- Calculation of Hotelling T^2 statistic
- One variable SPC chart Vs. multivariate chart
- Radar plots
- EXCEL capabilities

For further information

- Adams, B.M., "The Multivariate Control Web," *Quality Engineering*, 6(4), 533-545, 1994
- Montgomery, D.C., *Statistical Quality Control*, 2nd ed., Wiley, 1991
- Ryan, T.P., *Statistical Methods for Quality Improvement*, Wiley, 1989
- Tracy, N.D., Young, J.C., Mason, R.L., "Multivariate Control Charts for Individual Observations," *JQT*, 24(2), April 1992