An EXCEL Add-In for Comparing Two Exponential Distributions

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Reliability Analysis Problem

- Exponentially distributed times to failure
- Want to compare two
 - MTTFs (non-repairable components)

or

– MTBFs (repairable systems)

Typically censored data (Time or Count censoring)

Statement of Problem

- Two exponentially distributed data sets of <u>count</u> censored data
- Let T_i be the total time on test, i = 1,2

$$T_{i} = \sum_{j=1}^{r_{i}} t_{i(j)} + (n_{i} - r_{i}) t_{i(r_{i})}$$

The mean time to failure (MTTF) estimate is

$$\hat{\theta}_i = T_i / r_i$$

Distributions of Statistics

• T_i has a distribution given by

$$2T_i / \theta_i \sim \chi^2_{(2r_i)}$$

• The ratio

$$\frac{2T_1/2r_1\theta_1}{2T_2/2r_2\theta_2} = \frac{\hat{\theta}_1\theta_2}{\hat{\theta}_2\theta_1}$$

has an *F*-distribution with $(2r_1, 2r_2)$ degrees of freedom

Procedure

- Set up two-sided 100 γ % confidence interval for the ratio θ_1/θ_2
- Confidence limits are

$$\hat{\rho} = \left(\hat{\theta}_1 / \hat{\theta}_2\right) / F\left[\left(1 + \gamma\right) / 2; 2r_1; 2r_2\right]$$
$$\tilde{\rho} = \left(\hat{\theta}_1 / \hat{\theta}_2\right) \cdot F\left[\left(1 + \gamma\right) / 2; 2r_2; 2r_1\right]$$

• If interval **excludes** 1, then θ_1 and θ_2 differ statistically at the specified significance level

Example (Count Censored)

Two exponentially distributed data sets $\hat{\theta_1} = 12,600$ $\hat{\theta_2} = 8,800$ $r_1 = 16$ $r_2 = 22$ Ratio $(\hat{\theta_1}/\hat{\theta_2})$ 1.43 95% CI: LCL 0.76 UCL 2.80 Interval includes 1. Not significantly different.

Time Censored Data

The ratio

$$\frac{(1+0.5/r_{1})}{(1+0.5/r_{2})}\frac{\theta_{2}\hat{\theta}_{1}}{\theta_{1}\hat{\theta}_{2}}$$

has an *F*-distribution with $(2r_1+1, 2r_2+1)$ degrees of freedom.

Procedure

- Set up two-sided 100 γ % confidence interval for the ratio θ_1/θ_2
- Confidence limits are

$$\hat{\rho} = \frac{(\hat{\theta}_1 / \hat{\theta}_2)(1 + 0.5 / r_1)}{(1 + 0.5 / r_2)} / F[(1 + \gamma) / 2; 2r_1 + 1; 2r_2 + 1]$$

$$\tilde{\rho} = \frac{(\hat{\theta}_1 / \hat{\theta}_2)(1 + 0.5 / r_1)}{(1 + 0.5 / r_2)} \cdot F[(1 + \gamma) / 2; 2r_2 + 1; 2r_1 + 1]$$

• If interval **excludes** 1, then θ_1 and θ_2 differ statistically at the specified significance level

Example (Time Censored)

Two exponentially distributed data sets $\hat{\theta}_1 = 120$ $\theta_2 = 55$ $r_{2} = 15$ $r_{1} = 10$ Ratio $(\hat{\theta}_1/\hat{\theta}_2)$ 2.18 95% CI: LCL 1.03 UCL 5.10 Interval excludes 1. Significantly different.

EXCEL Add-In

 This visual basic add-in makes the comparison of exponential distributions a snap!

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4				Group 1				Type of Cen	soring		
5				Enter MTB	F for Group 1			Count Censored			
6							C Time Censored				
7				12600				- THE C	risoren		
8											
9				Enter Number of Failures for Group 1			Output: Reference				
10				16				\$4\$1	2		
11											
12				- Group 2				Confidence L	evel (example	0.95)	
1.3				Enter MTB	F for Group 2	,		0.95	_		
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Software Example

Simply Execute Our User-friendly Macro By Using the Pull Down Menu Provided.

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Software Example Cont.

•Select the Censoring Type.

•Fill In The MTBF and Number of Failures for Each Distribution, as shown below.

Compare Two Exponential Distributions	? ×
Group 1	Type of Censoring
Enter MTBF for Group 1	Count Censored
12600	C Time Censored
Enter Number of Failures for Group 1	Output Reference
16	\$A\$1
Group 2	Confidence Level (example 0.95)
Enter MTBF for Group 2	0.95
8800	
Enter Number of Failures for Group 2	
22	OK

Software Example Cont.

Click the "OK" Button, and Let the Software Do the Rest. We Do All of the Calculations For You!

Microsoft Excel - Book1									
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	A1 = Compare Two MTBF Values (Count Censored Data)								
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1	Compare Two MTBF Values (Count Censored Data)								
2									
3	MTBF 1	12600							
4	Number of Failures in Group 1	16							
5	MTBF 2	8800							
6	Number of Failures in Group 2	22							
7									
8	Confidence Level	0.95							
9									
10	Ratio of MTBF 1 to MTBF 2	1.431818							
11	Lower Confidence Level	0.757154							
12	Upper Confidence Level	2.801486							
13									
1.4									

Summary

- Simple procedures provided to compare two exponential distributions
- Excel add-in availability at www.trindade.com/mttf.htm

References

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