A GENERAL EXCEL SOLUTION FOR LTPD TYPE SAMPLING PLANS

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Abstract

In this paper we discuss a fairly common problem in lot acceptance sampling. Suppose a company qualification or lot acceptance plan calls for a large sample size n with a non-zero acceptance number c. For example, a plan may call for accepting a lot on three or less failures out of 300 sample units. Because the sample units are costly, the manufacturer wants to reduce the acceptance number and consequently the sample size while holding the rejectable quality percent defective p value constant at a specified consumer's risk (e.g., 10% probability of acceptance). Thus, an LTPD sampling plan is desired. However, typical tables available in the literature cover only a limited number of p values for a few risk probabilities. For the general situation of any desired p value at any probability of acceptance, what are the sample sizes n corresponding to various c values? We describe methods for obtaining solutions via EXCEL and present a simple add-in for the calculations.

Introduction

There are several ways to categorize a single-sampling lot acceptance plan for fraction non-conforming product. See discussion in Western Electric handbook on acceptance sampling [1]. In particular, plans that have a low probability of acceptance (say, 10%) for a specified quality level are called LTPD or Lot Tolerance Percent Defective plans. This quality level is considered the highest percent defective (that is, the poorest quality) that can be tolerated in a small percentage of the product. The Dodge-Romig [2] Sampling Inspection Tables define LTPD as "an allowable percent defective which may be considered as the borderline of distinction between a satisfactory lot and an unsatisfactory one." An alternate name for the LTPD is the rejectable quality level or RQL.

For our purposes, we will consider a random sample of a lot from a process or from a large lot for which the sample is less than 10% of the lot size (Type B sampling). In this manner, we can apply the binomial distribution exactly instead of the more complicated hypergeometric distribution (Type A). For a discussion of the differences between Type A and B sampling see Duncan [3] or Montgomery [4].

The probability of acceptance of the poorest quality LTPD that can be tolerated in an individual lot is often referred to as the "consumer's risk." A value of 10% for the consumer's risk is commonly referenced in LTPD plans. Many product qualification plans are based on LTPD considerations in order to assure consumer protection against individual lots of poor quality. However, a common problem is the need to adjust the sample lot size and corresponding acceptance number based on time, money, or resource considerations while holding the consumer's risk constant at a specified quality level. Because of this objective, many tables are available (see [2], [5] or [6]) to assist in the proper selection of sample sizes and acceptance numbers. A difficulty with utilizing these tables is the restriction to only the values specified in the tables. Other authors have provided graphical solutions to handle more general requirements. See Tobias and Trindade [7]. Further details on LTPD schemes can be found in Schilling [8]. We present below a general EXCEL solution that handles any specified LTPD at any probability of acceptance.

The following example is based on the problem (solved graphically) in Tobias and Trindade [7]

Example 1: Consider the case where a given sampling plan calls for a lot to be accepted on three or less failures out of 300 sampled units. Because of the cost associated with each individual unit, we need to construct a sampling plan that will allow the lot to be rejected on just one failure. One important requirement of the new plan is that the LTPD and the probability of acceptance be equivalent to that of the original plan. In this example, we will set the probability of acceptance equal to 10%.

Step 1: Begin by creating in a spreadsheet a table like the one shown in Figure 1, specifying the sample size n, the acceptance number c, and the probability of acceptance (10% for LTPD plans). We are interested in determining the percent defective at the specified probability of acceptance, that is, the LTPD.

Step 2: The binomial cumulative distribution function gives the probability of realizing up to c rejects in a sample of size n. To determine the percent defective value that makes the binomial CDF equal to the probability of acceptance value, we use an identity between the binomial CDF $F(x)$ and the Beta CDF $G(p)$ for integer valued shape parameters. (See Bury [9], page 346.) The relationship is:

$$F(x;\ p,\ n) = 1 - G(p;\ x + 1,\ n - x).$$

We use the EXCEL BETAINV function to calculate the LTPD for the original plan, as illustrated in cell B10. The LTPD calculated in this example is 0.02213, as shown in Figure 2. The general format of the BETAINV is

BETAINV(1-prob of acceptance, c + 1, n – c).

Figure 1: Illustration of how the Excel worksheet should be set up prior to solving for a solution to the original sampling plan.

Figure 2: Illustration of LTPD calculated for original plan where c=3, n=300, and prob of acceptance = 0.1.

Step 3: Create a second table with two columns. The first column should contain a list of possible sample sizes in descending order if a lower acceptance number is desired or in ascending order for a higher c.

Figure 3: Illustration of how the Excel worksheet should be set up prior to solving for the alternative sample size.

The second column should contain the EXCEL BINOMDIST function (binomial CDF) for each sample size, LTPD, acceptance number combination, as shown in Figure 3 (column F). Our goal is to find the smallest value of n that yields a BINOMDIST function value less than or equal to the probability of acceptance, 10%. From Figure 4 we can easily see that the appropriate sample size is 175. This is the smallest possible sample size for which the BINOMDIST function is less than or equal to 0.10. The format of the BINOMDIST function is

BINOMDIST(acceptance number, sample size, LTPD, 1)

The final "1" instructs Excel to calculate the cumulative distribution function.

Figure 4: Illustration of solving for the correct value of n.

The new sampling plan calls for rejection of any lot where more than 1 unit fails out of a sample of 175 units. The probability of accepting any given lot at an incoming percent defective value equal to the LTPD is 10%.

Automated Analysis Tools

The process of finding an alternative LTPD sampling plan can be made simple through the use of visual basic macros. The example below introduces a user-friendly Excel macro that performs LTPD sampling plan calculations. We will use the data from the previous example where $c = 3$, $n = 300$, and probability of acceptance = 10%.

Step 1: Execute the macro and select "Find alternative sampling plan." See Figure 5.
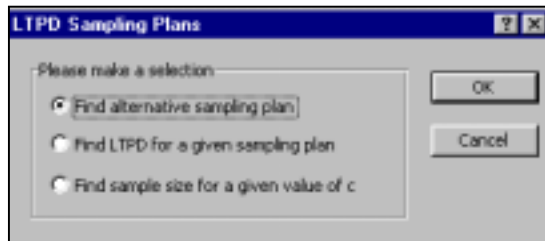

Figure 5: Illustration of LTPD sampling plans dialog box.

Step 2: Enter the information for the original sampling plan. Then, enter a new acceptance number for the alternative sampling plan. Finally, enter the worksheet cell address where output table should be placed. Figure 6 illustrates typical input.
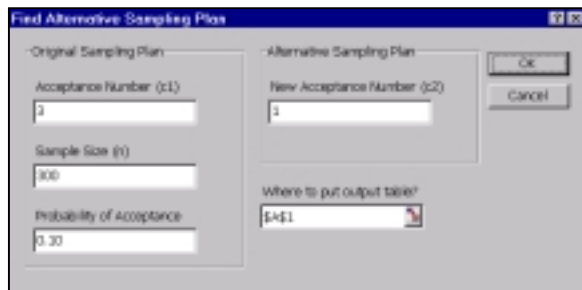

Figure 6: Illustration of alternative sampling plan dialog box.

Step 3: Click the "OK" button. The final analysis table is show in Figure 7. The routine also provides the approximate AQL to accept 95% of the lots.

The program also provides the capability for: 1. solving for the LTPD for a given sampling plan; 2. solving for the sample size for a specified LTPD and acceptance number. The execution of these routines are self-evident from the dialog boxes.

Figure 7: Illustration of the alternative LTPD sampling plan output table. The sample size for the alternative sampling plan is 175 (reference cell C10).

Program Availability
The EXCEL LTPD add-in, written in Visual Basic by David Meade, is available for free downloading from the website www.trindade.com/LTPD.html.

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