“Failure Rate” Which One ?

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The confusion

- The term failure rate is used to express the reliability of non-repairable components.
- It is used to express the reliability of repairable systems.
- It is used to express the reliability of a non-repairable component functioning inside a repairable system.
- The meaning of the term failure rate is different in each of these contexts.
- Improper analysis techniques are often used due to this misunderstanding.

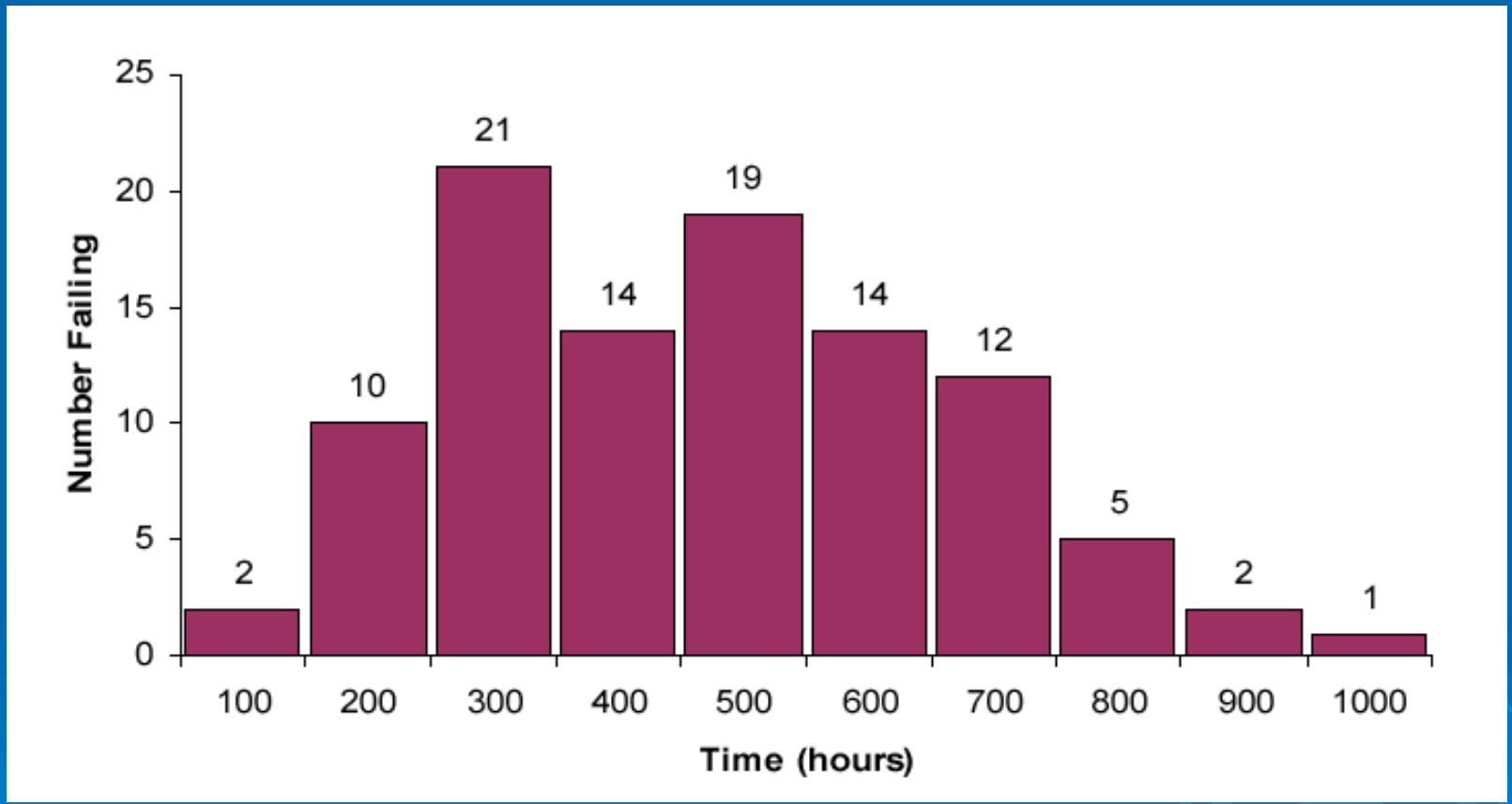
Non-Repairable System

- A system (component, unit, part, etc.) that is discarded upon failure
  - It is not repaired.
  - The lifetime is a random variable *T* described by a **single** time to failure.
  - For a group of identical systems, the lifetimes are assumed independent and come from a single population.
  - We say the lifetimes are “independent and identically distributed ”.

Non-Repairable System Example

- 100 microprocessors are put into an oven for a dynamic stress (voltages varied) at an elevated temperature.
- The planned test duration is 1000 hours.
- Units are removed from the ovens and tested every 100 hours (readout date). Non-failed units are returned to stress.
- The number of units failing in each 100 hour interval is shown as a histogram.

Failure Frequency in each 100 hour interval

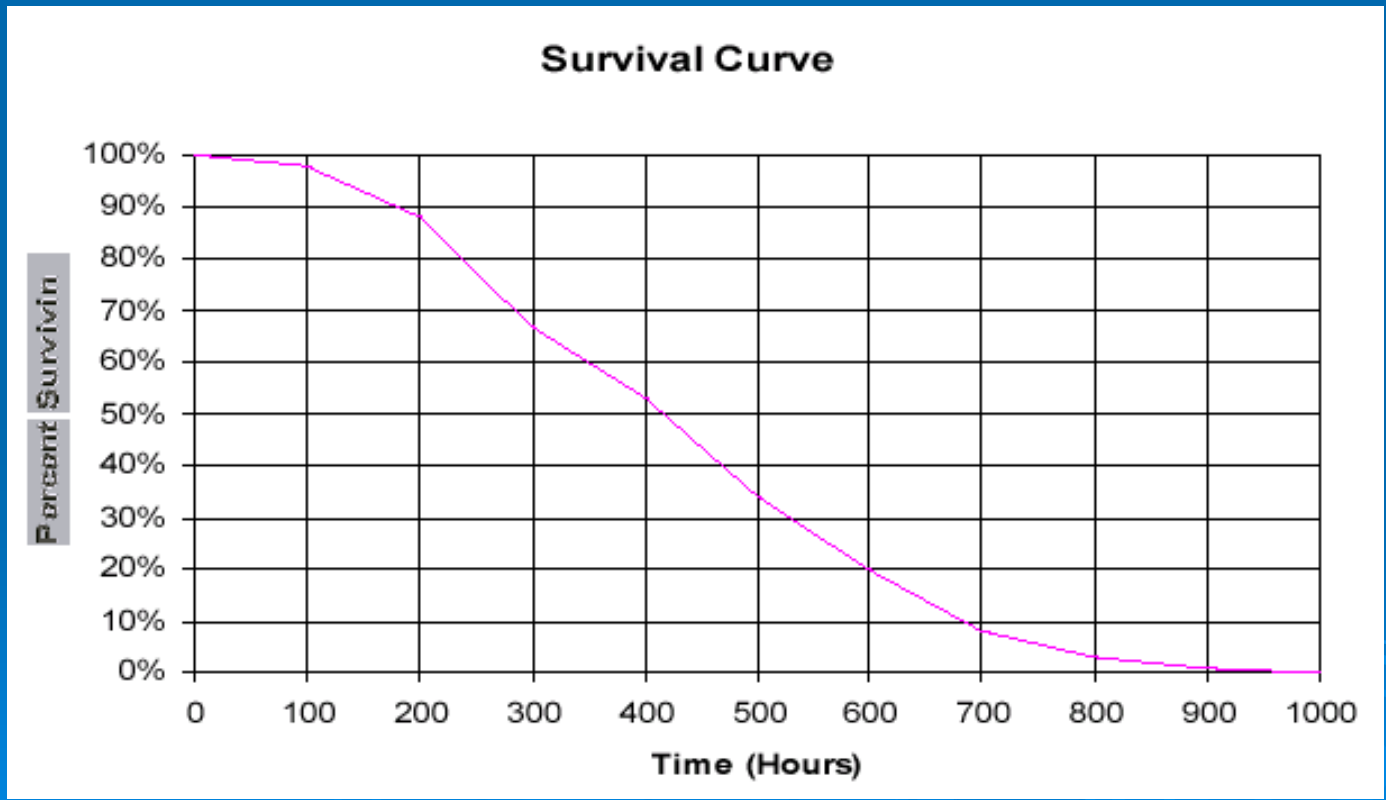


Interpreting the Histogram

- The number of failures in each 100 hour interval increases and then decreases.
- Does this mean the failure rate is also increasing and then decreasing?
- Why does the number of failures start decreasing for later intervals?

Survival Curve

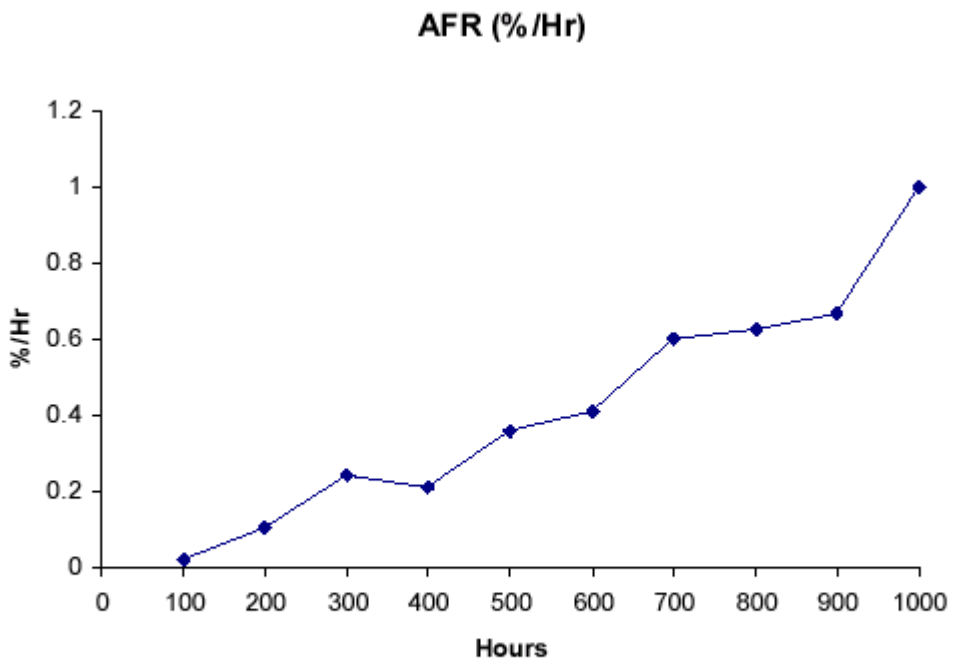
- As failures occur, the sample size on stress decreases.
- The percent of units surviving at the end of each 100 hour readout is shown as a survival curve.



Estimating Failure Rate

- Divide the number of units that fail **during** an interval (see histogram) by the number of units alive at the **beginning** of the interval (see survival curve) to obtain the fraction failing during that interval.
- Divide that fraction by the length of the interval (100 hours) to estimate the average “failure rate”, *AFR*, during that interval.

Average “Failure Rate” During Intervals



Average failure rate is clearly increasing.

Failure Rate = Hazard Rate

- In the limit of smaller time intervals, the average “failure rate” measures the rate of failure in the next instant on time for those units (conditioned on) surviving to time t: called the instantaneous failure rate, *IFR*.
- AFR and IFR are “hazard” rates. Also called the force of mortality. These “failure rates” are a property of a time to failure distribution.

Repairable System

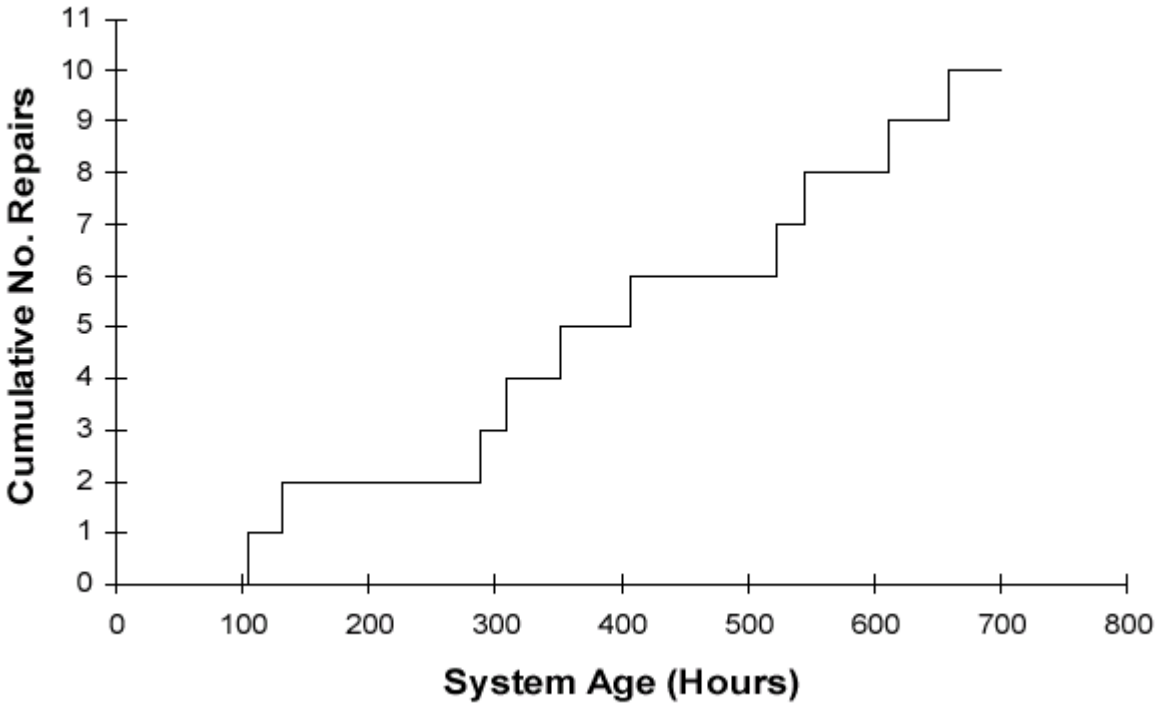
- A repairable system is a system which, upon failure, is restored to operation by any repair action other than replacing the entire system.
- The lifetime of the system is the **age** T of the system, that is, the total hours of operation.

Random Variables for Repairable Systems

- There are two random variables of interest
  - The **number** of failures $N(t)$ by age t
  - The **time** between successive failures X_i .
- The **age** of the system at the ith failure T_i is given by

$$T_i = X_1 + X_2 + \cdots + X_i$$

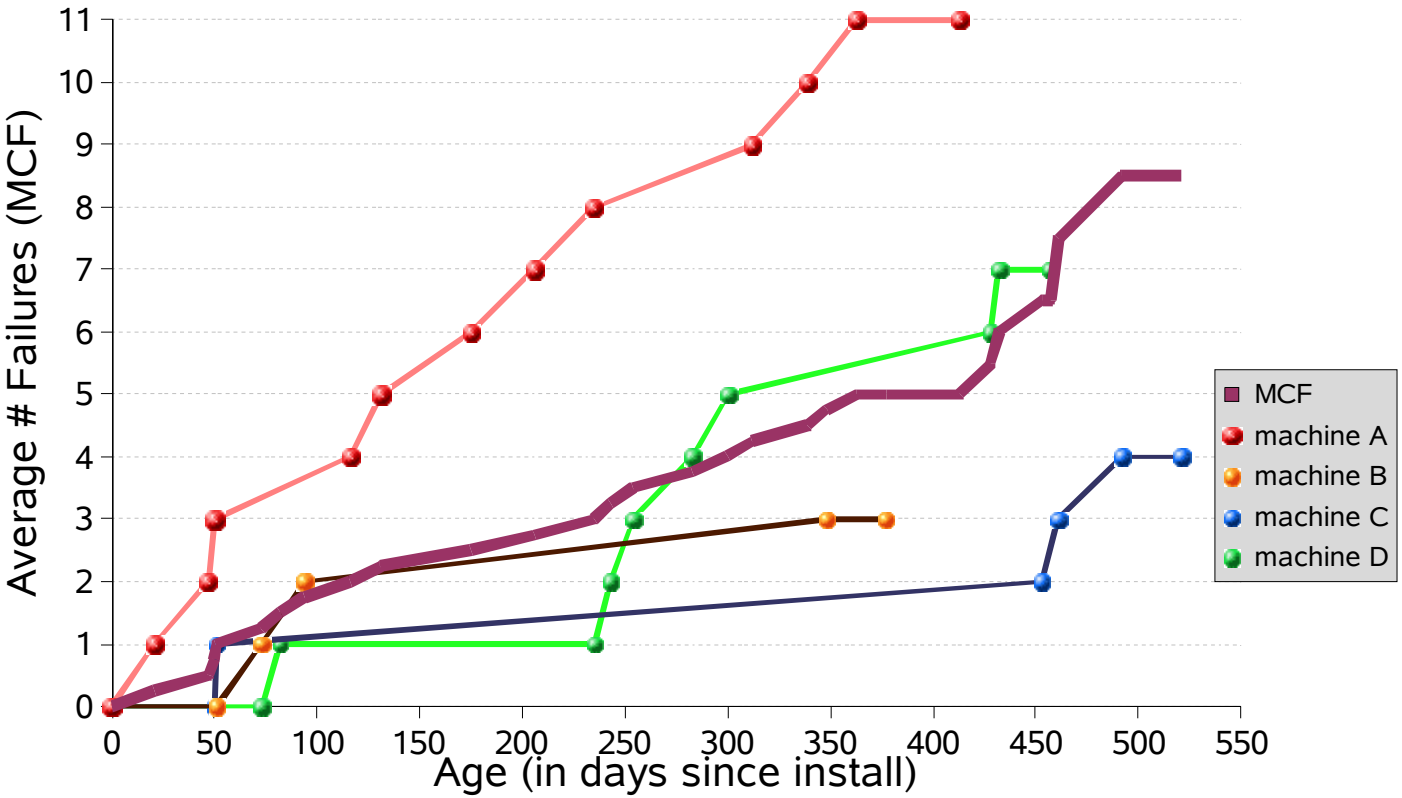
Cumulative Plot : $N(t)$ vs. t



Mean Cumulative Function

- Consider a group of machines with individual cumulative plots (failure histories), $N_i(t)$.
  - MCF represents the average behavior of the cumulative plots across a group of systems at risk at any point in time.
  - MCF can be viewed as a vertical slice across the individual cumulative plots at a time point, prior to any censoring.
  - MCF is the **average number of failures** of a group of systems at a particular age.

MCF and Cumulative Plots



Recurrence Rate or ROCOF

- For repairable systems, the “failure rate ” is the **rate of occurrence** of failures (ROCOF) or simply the recurrence rate *RR*, versus the system age.
- The *RR* is the derivative *m*(*t*) of the MCF and is also called the intensity function.
- The intensity function is the probability (unconditional) of failure in a small interval of time divided by the length of the interval.
- This *RR* can be quite different from the hazard rate as it is a property of a ***sequence of failure times*** as opposed to a property of a ***single time to failure distribution.***

ROCOF vs Hazard Rate

- ROCOF (rate of occurrence of failures) is the probability that a failure (not necessarily the first) occurs in a small time interval.
- Hazard rate is the conditional probability that a component fails in a small time interval given that it has survived from time zero until the beginning of the time interval.
- ROCOF is the *absolute* rate at which a system failures occur
- Hazard rate is the *relative* rate of failure of components *surviving until time T.*

Example Data

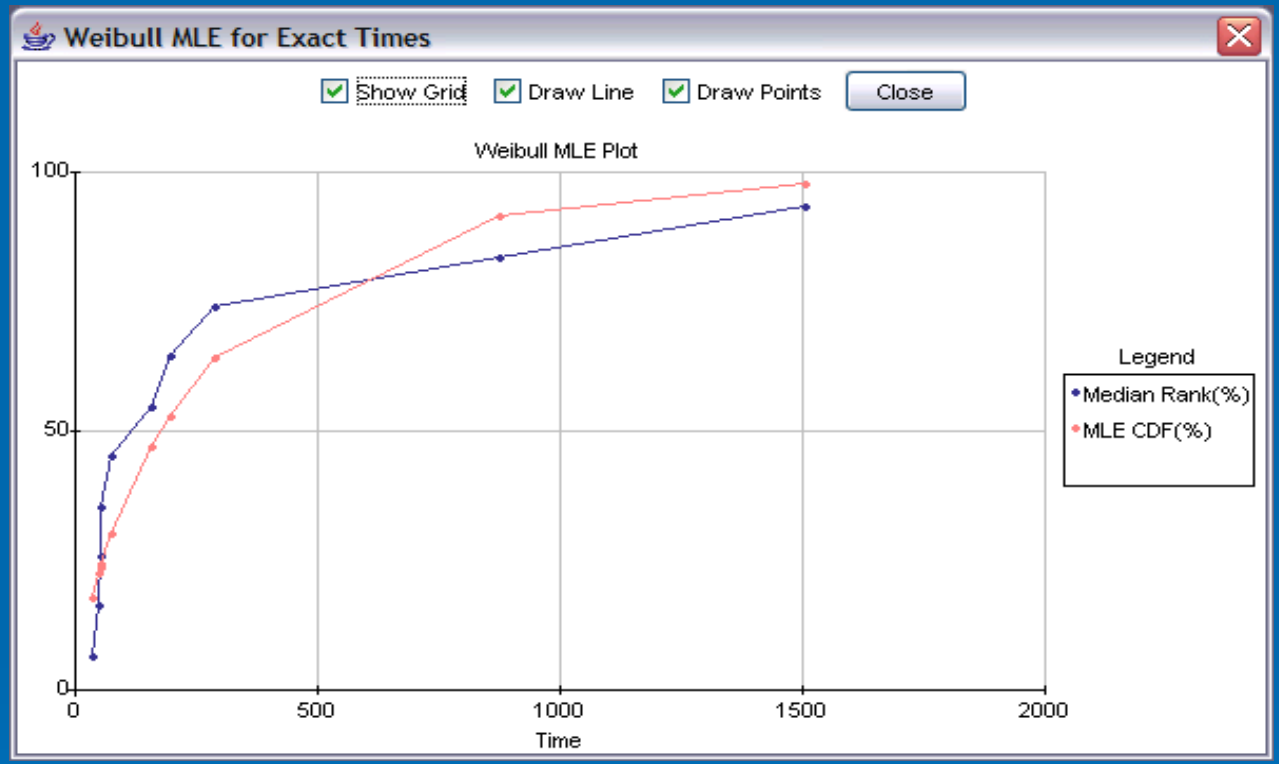
Failure Number	*Failure Times*	*Time Between Failures*
1	876	876
2	2382	1506
3	2576	194
4	2863	287
5	2912	49
6	2964	52
7	3120	156
8	3195	75
9	3249	54
10	3284	35

- The data describes the times to failure of a server due to a **single** component X.
- Component X is replaced at every failure time.
- The times between failures are how long each new component X lived in the system

Non-Repairable Systems Approach

- The times between failures are treated as the lifetimes of component X.
- The lifetimes can be sorted by magnitude.
- One can do distributional fitting on these ordered times to failure.
- There is no difference between component X being replaced ten times within a system compared to ten components being placed on a life test.
- Both testing methods are assumed to provide equivalent data, and the order of failures does not matter.

Weibull Fitting



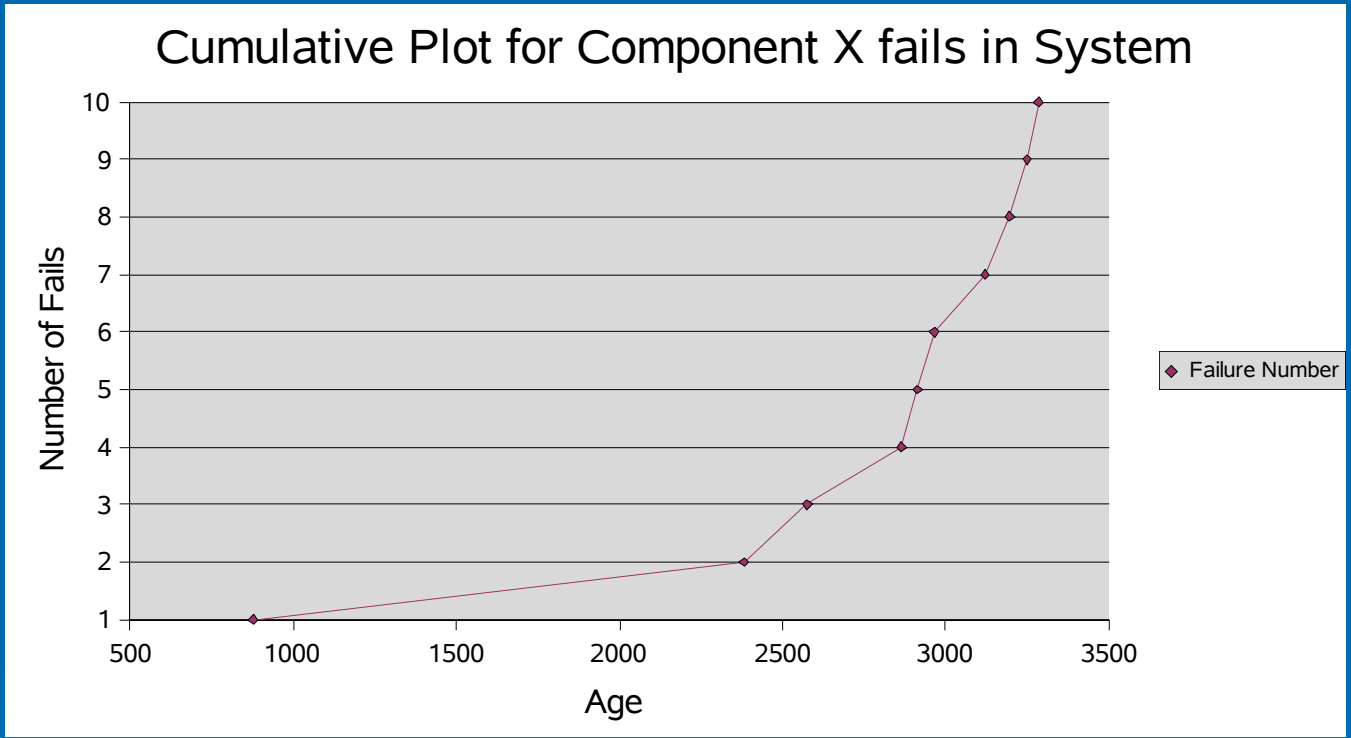
MLE Estimates :

Characteristic Life = 277.0 hours

Shape Parameter = 0.78

Shape parameter < 1 → decreasing hazard rate

Repairable Systems Approach



- A plot of number of fails versus age.
- Failures are occurring more frequently as the system ages.
- Rate of occurrence of failures ROCOF or recurrence rate (RR) is actually increasing!

What is really happening ?

- Times between failures are not independent and identically distributed, i.e, time to first failure distribution is not the same as time between first and second failure.
- Order of occurrence of failures is important because components are within a repairable system.
- A degrading fan resulting in poor cooling of component X caused the increased rate of failures.

Wrong Analysis = Wrong Conclusion

- Non-repairable analysis: sorting the times between failures loses the order of occurrence of failures.
- Repairable system analysis: retaining the order of occurrence of failures reveals any trends in system behavior.
- Using the term “failure rate” in both cases adds to the confusion.

Summary

- **Hazard** rates apply to non-repairable systems.
- **ROCOF** or recurrence rates ***RR*** apply to repairable systems.
- The generic term “failure rate” can lead to improper analysis because hazard rate and recurrence rates are conceptually different.
- For a repairable system, the recurrence rate can be increasing even if the replacement components come from a population with decreasing hazard rates or vice versa.