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PROPAGATION OF ERROR IN ACCELERATED STRESSING

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ABSTRACT

In an acceleration equation, uncertainties may exist in the empirically determined constants of the model. This report reviews certain points of the theory of error and discusses the relation to acceleration models with examples in electromigration.

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INTRODUCTION

Because of time limitations, reliability experiments are often conducted at elevated stress levels to produce failures sooner than would result at normal conditions. For example, a higher temperature or voltage or the combination of both might be employed. The data obtained is then used to project the expected behavior at usual or "field" operation.

If sufficient theoretical or experimental work has generated a model describing the influence of the applied stresses on the time to fail, then the model is utilized in estimating parameters of the field distribution. It is important, however, that the propagation of uncertainty involved in applying a model be understood. This report briefly reviews the theory of error and considers the use of accelerated data for projecting field expectations for some examples in electromigration.

THEORY

The acceleration factor (AF) is defined as the ratio of the times necessary to achieve any given failure probability, providing the failure distributions are of the same type. In terms of the cumulative distribution function F, the definition states, for times t, that

$$AF = \left(\frac{t_2}{t_1}\right)_{F_1 = F_2}$$

or simply that

$$t_2 = AF \times t_1 \tag{1}$$

The desired probability may be any value but it is common to select the fifty percent point in modeling with the lognormal distribution.

According to the theory for the propagation of uncertainties [1], if Z is a function of two variables, x and y, that is

$$Z = f\ (\ x,y\ )$$

and we are interested in the uncertainty δZ in Z resulting from uncertainties δx and δy in x and y, respectively, then

$$\delta Z = \left(\frac{\partial f}{\partial x}\right) \delta x + \left(\frac{\partial f}{\partial y}\right) \delta y \qquad (2)$$

where the derivatives are evaluated at the x and y values at which Z is required. We note that the uncertainties are defined [1] as "outer limits of confidence within which we are 'almost certain' (i.e., perhaps 99 percent certain) that the measurement lies". If sufficient measurements have established the standard deviation S_x and S_y of the x and y values, then it is possible [2] to calculate via the theory of error the standard deviation S_Z of Z. For example, for independent uncertainties δx, δy the equation would be

$$S_Z = \left[(\partial f/\partial x)^2 S_x{}^2 + (\partial f/\partial y)^2 S_y{}^2 \right]^{1/2}$$

However, for most cases in modeling the parameters are estimated from limited, indeed even censored, experiments so that the bounds on the independent variables are expressible more in terms of the first definition than in statistical terms.

Thus, applying equation (2) to (1) results in the uncertainty of t_2 as

$$\delta t_2 = \left(\frac{\partial t_2}{\partial AF}\right) \delta AF + \left(\frac{\partial t_2}{\partial t_1}\right) \delta t_1 \qquad (3)$$

where δAF and δt_1 are the uncertainties in the acceleration factor and the time t_1, respectively.

Now

$$\left(\frac{\partial t_2}{\partial AF}\right) = t_2$$

and

$$\left(\frac{\partial t_2}{\partial t_1}\right) = AF$$

So, (3) becomes

$$\delta t_2 = t_2 \delta AF + AF \delta t_1 \tag{4}$$

A more useful quantity may be the relative uncertainty (R.U.) given by dividing (4) by (1). This procedure gives

$$\frac{\delta t_2}{t_2} = \frac{\delta AF}{AF} + \frac{\delta t_1}{t_1} \tag{5}$$

that is the R.U. of t_2 is equal to the sum of the R.U.'s of the terms forming the product. This relationship is extendible to any number of first order terms.

Equation (5) shows that the R.U. of a product can never be less than the R.U. of one of the individual terms. In particular, if there is a large degree of uncertainty in the acceleration factor as a result of uncertainty in the parameters making up the model, the projected time will also have considerable error.

EXAMPLES

The techniques we show are general and can be applied to any model to investigate the influence of error. However, we will investigate the propagation of error for the specific case of electromigration. This phenomenon, in which current-carrying conductors will open or fail in time as a result of the mass transport of metal ions in an electric field, has been modeled [3].

The Chhabra - Ainslie model for electromigration states that the time to failure is

$$t_f = A\,J^{-n}\exp[\Delta H/KT] \tag{6}$$

where

A = a constant,

J = current density,

N = a positive number between 1 and 3 depending on physical and material constraints,

ΔH = an appropriate activation energy,

KT = product of Boltzman's constant, and absolute temperature.

For temperature gradient failure, n is theoretically 3. For electromigration at a barrier to diffusion, n equals 1 theoretically.

However, Black [4] in a discussion of the physics of electromigration cites experimental situations where the current exponent n has ranged between 1 and 3. Similarly he reports that the value for the activation energy has ranged between 0.48 eV to 1.2 eV depending on, for example, the grain boundary properties of the metal film.

Hence, because of the possible variation in values, it is common practice to experimentally determine values for n and ΔH for the particular product undergoing testing or qualification for field use. We will use the theory presented to illustrate the influence of uncertainties in n and ΔH on the acceleration factor.

From equations (2) and (6), we write the acceleration factor as

$$AF = \left(\frac{J_1}{J_2}\right)^n \exp \frac{\Delta H}{K}\left[\left(\frac{1}{T_2} - \frac{1}{T_1}\right)\right] \qquad (7)$$

where J_1 are the current densities and T_1 are the absolute temperatures for the particular stress or use condition. Let us write the equation (7) as

$$AF = f\ (\ n, \Delta H\ ) \qquad (8)$$

where we assume for simplicity that the current densities and temperatures are values with no error or uncertainty. This assumption may or may not be reasonable depending on the experimental setup, but the procedure can be expanded to include the uncertainties if desired. However, for purposes of illustration only, we shall treat the simple case.

The uncertainty in AF can now be expressed through equation (2) as

$$\delta AF = \left(\frac{\partial AF}{\partial n}\right) \delta n + \left(\frac{\partial AF}{\partial(\Delta H)}\right) \delta(\Delta H) \qquad (9)$$

where δn and $\delta(\Delta H)$ are the uncertainties (i.e., outer limits of confidence) in the current exponent and activation energy, respectively. Now, from the model (7)

$$\left(\frac{\partial AF}{\partial n}\right) = \left(\frac{J_1}{J_2}\right)^n \left[\ell n \left(\frac{J_1}{J_2}\right)\right] \exp\left[\frac{\Delta H}{K}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)\right]$$

or

$$\left(\frac{\partial AF}{\partial n}\right) = AF \;\ell n \left(\frac{J_1}{J_2}\right)$$

Similarly, we get

$$\left(\frac{\partial AF}{\partial(\Delta H)}\right) = \left(\frac{J_1}{J_2}\right)\left[\frac{1}{K}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)\right] \exp\left[\frac{\Delta H}{K}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)\right]$$

$$= AF\left[\frac{1}{K}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)\right]$$

Thus, the R. U. of AF becomes

$$\frac{\delta AF}{AF} = \delta n\left(\ell n \frac{J_1}{J_2}\right) + \delta(\Delta H)\left[\frac{1}{K}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)\right] \qquad (10)$$

Equation (10) indicates that the relative uncertainty in the acceleration factor is independent of n and ΔH. The value depends only on the ratio of the currents, the difference in the reciprocal absolute temperatures, and the magnitudes of the uncertainties (absolute, not relative) δn and $\delta(\Delta H)$.

The percentage magnitudes for the R. U. of AF as calculated by equation (10) for several sets of current ratios and temperatures are shown in Tables 1 through 11. We read, for example, from Table 2 for $\delta n = \pm .2, \delta(\Delta H) = \pm .05$, $J_1/J_2 = 5$, and $T_1 = 150°C$, $T_2 = 35°C$ that the uncertainty in the acceleration factor is $\pm$ 83%.

Given the R.U. of the AF via equation (10), we can apply equation (5) to estimate the R.U. of the projected time t_2 for a given R.U. of the measured time t. Thus, if the experimental cell has, for example, a time to fifty percent fail, that is t_1, which is uncertain to 20% for the previous set of conditions, the relative uncertainty in the t50 of the lower stress cell will be 83% + 20% or approximately 100%.

REFERENCES

[1] D. C. Baird, Experimentation (Prentice-Hall, Englewood, N.J., 1962).

[2] Yardley Beers, Introduction to the Theory of Error (Addison-Wesley, Reading, Mass., 1957).

[3] D. Chhabra and N. Ainslie, IBM SPD Division, East Fishkill, N. Y., "Open Circuit Failures in Thin Film Conductors," Tech. Report 22.419, July 1967.

[4] James R. Black, Motorola Semiconductor Product Division, Phoenix, Physics of Electromigration, Proceedings of the Reliability Physics Symposium, Las Vegas, Nev., 1974.

APPENDIX

TABLES

The tables shown are for the following conditions.

Table 1	T_1=150°c, T_2=35°c,	J_1/J_2=2.5
2		J_1/J_2=5
3		J_1/J_2=10
4	T_1=150°c, T_2=50°c,	J_1/J_2=2.5
5		J_1/J_2=5
6		J_1/J_2=10
7	T_1=150°c, T_2=85°c,	J_1/J_2=2.5
8		J_1/J_2=5
9		J_1/J_2=10
10	T_1=125°c, T_2=50°c,	J_1/J_2=5
11	T_1=125°c, T_2=50°c,	J_1/J_2=100

Table 1

ENTER AS A VECTOR THE ABSOLUTE UNCERTAINTY IN N:

⎕:

.1 .2 .3 .5 1

ENTER AS A VECTOR THE ABSOLUTE UNCERTAINTY IN ΔH:

⎕:

.01 .02 .05 .1 .25

ENTER THE FIELD AND STRESS TEMPERATURES (∘C) AS A VECTOR:

⎕:

35 150

ENTER THE RATIO OF STRESS TO FIELD CURRENT DENSITIES:

⎕:

2.5

THE MATRIX OF RELATIVE UNCERTAINTIES OF THE ACCELERATION FACTOR, THAT IS, (ΔAF)÷AF IN PERCENT, FOR ABSOLUTE UNCERTAINTIES (A.U.) IN "ΔH", I.E. COLUMNS, OR IN "N", I.E. ROWS, IS:

A.U.(ΔH)=		.010	.020	.050	.100	.250
A.U.(N)=	.10	19.4	29.6	60.3	111.5	264.9
	.20	28.6	38.8	69.5	120.6	274.1
	.30	37.7	48.0	78.6	129.8	283.3
	.50	56.0	66.3	97.0	148.1	301.6
	1.00	101.9	112.1	142.8	193.9	347.4

Table 2

ENTER AS A VECTOR THE ABSOLUTE UNCERTAINTY IN N:

⎕:

.1 .2 .3 .5 1

ENTER AS A VECTOR THE ABSOLUTE UNCERTAINTY IN ΔH:

⎕:

.01 .02 .05 .1 .25

ENTER THE FIELD AND STRESS TEMPERATURES (∘C) AS A VECTOR:

⎕:

35 150

ENTER THE RATIO OF STRESS TO FIELD CURRENT DENSITIES:

⎕:

5

THE MATRIX OF RELATIVE UNCERTAINTIES OF THE ACCELERATION FACTOR, THAT IS, (ΔAF)÷AF IN PERCENT, FOR ABSOLUTE UNCERTAINTIES (A.U.) IN "ΔH", I.E. COLUMNS, OR IN "N", I.E. ROWS, IS:

A.U.(ΔH)=		.010	.020	.050	.100	.250
A.U.(N)=	.10	26.3	36.6	67.2	118.4	271.9
	.20	42.4	52.7	83.3	134.5	288.0
	.30	58.5	68.7	99.4	150.6	304.1
	.50	90.7	100.9	131.6	182.8	336.2
	1.00	171.2	181.4	212.1	263.3	416.7

Table 3

ENTER AS A VECTOR THE ABSOLUTE UNCERTAINTY IN N:

⎕:

.1 .2 .3 .5 1

ENTER AS A VECTOR THE ABSOLUTE UNCERTAINTY IN ΔH:

⎕:

.01 .02 .05 .1 .25

ENTER THE FIELD AND STRESS TEMPERATURES (∘C) AS A VECTOR:

⎕:

35 150

ENTER THE RATIO OF STRESS TO FIELD CURRENT DENSITIES:

⎕:

10

THE MATRIX OF RELATIVE UNCERTAINTIES OF THE ACCELERATION FACTOR, THAT IS, (ΔAF)÷AF IN PERCENT, FOR ABSOLUTE UNCERTAINTIES (A.U.) IN "ΔH", I.E, COLUMNS, OR IN "N", I.E. ROWS, IS:

A.U.(ΔH)=		.010	.020	.050	.100	.250
A.U.(N)=	.10	33.3	43.5	74.2	125.3	278.8
	.20	56.3	66.5	97.2	148.4	301.8
	.30	79.3	89.5	120.2	171.4	324.8
	.50	125.4	135.6	166.3	217.4	370.9
	1.00	240.5	250.7	281.4	332.6	486.0

Table 4

ENTER AS A VECTOR THE ABSOLUTE UNCERTAINTY IN N:

⎕:

.1 .2 .3 .5 1

ENTER AS A VECTOR THE ABSOLUTE UNCERTAINTY IN ΔH:

⎕:

.01 .02 .05 .1 .25

ENTER THE FIELD AND STRESS TEMPERATURES (∘C) AS A VECTOR:

⎕:

50 150

ENTER THE RATIO OF STRESS TO FIELD CURRENT DENSITIES:

⎕:

2.5

THE MATRIX OF RELATIVE UNCERTAINTIES OF THE ACCELERATION FACTOR, THAT IS, (ΔAF)÷AF IN PERCENT, FOR ABSOLUTE UNCERTAINTIES (A.U.) IN "ΔH", I.E. COLUMNS, OR IN "N", I.E. ROWS, IS:

A.U.(ΔH)=		.010	.020	.050	.100	.250
A.U.(N)=	.10	17.6	26.1	51.6	94.0	221.2
	.20	26.8	35.3	60.7	103.2	230.4
	.30	36.0	44.5	69.9	112.3	239.6
	.50	54.3	62.8	88.2	130.6	257.9
	1.00	100.1	108.6	134.0	176.5	303.7

Table 5

ENTER AS A VECTOR THE ABSOLUTE UNCERTAINTY IN N:

⎕:

.1 .2 .3 .5 1

ENTER AS A VECTOR THE ABSOLUTE UNCERTAINTY IN ΔH:

⎕:

.01 .02 .05 .1 .25

ENTER THE FIELD AND STRESS TEMPERATURES (°C) AS A VECTOR:

⎕:

50 150

ENTER THE RATIO OF STRESS TO FIELD CURRENT DENSITIES:

⎕:

5

THE MATRIX OF RELATIVE UNCERTAINTIES OF THE ACCELERATION FACTOR, THAT IS, (ΔAF)÷AF IN PERCENT, FOR ABSOLUTE UNCERTAINTIES (A.U.) IN "ΔH", I.E. COLUMNS, OR IN "N", I.E. ROWS, IS:

A.U.(ΔH)=		.010	.020	.050	.100	.250
A.U.(N)=	.10	24.6	33.1	58.5	100.9	228.2
	.20	40.7	49.2	74.6	117.0	244.3
	.30	56.8	65.2	90.7	133.1	260.4
	.50	89.0	97.4	122.9	165.3	292.6
	1.00	169.4	177.9	203.4	245.8	373.0

Table 6

ENTER AS A VECTOR THE ABSOLUTE UNCERTAINTY IN N:

⎕:

.1 .2 .3 .5 1

ENTER AS A VECTOR THE ABSOLUTE UNCERTAINTY IN ΔH:

⎕:

.01 .02 .05 .1 .25

ENTER THE FIELD AND STRESS TEMPERATURES (•C) AS A VECTOR:

⎕:

50 150

ENTER THE RATIO OF STRESS TO FIELD CURRENT DENSITIES:

⎕:

10

THE MATRIX OF RELATIVE UNCERTAINTIES OF THE ACCELERATION FACTOR, THAT IS, (ΔAF)÷AF IN PERCENT, FOR ABSOLUTE UNCERTAINTIES (A.U.) IN "ΔH", I.E. COLUMNS, OR IN "N", I.E. ROWS, IS:

A.U.(ΔH)=		.010	.020	.050	.100	.250
A.U.(N)=	.10	31.5	40.0	65.4	107.9	235.1
	.20	54.5	63.0	88.5	130.9	258.1
	.30	77.6	86.0	111.5	153.9	281.2
	.50	123.6	132.1	157.5	200.0	327.2
	1.00	238.7	247.2	272.7	315.1	442.3

Table 7

ENTER AS A VECTOR THE ABSOLUTE UNCERTAINTY IN N:

⎕:

.1 .2 .3 .5 1

ENTER AS A VECTOR THE ABSOLUTE UNCERTAINTY IN ΔH:

⎕:

.01 .02 .05 .1 .25

ENTER THE FIELD AND STRESS TEMPERATURES (∘C) AS A VECTOR:

⎕:

85 150

ENTER THE RATIO OF STRESS TO FIELD CURRENT DENSITIES:

⎕:

2.5

THE MATRIX OF RELATIVE UNCERTAINTIES OF THE ACCELERATION FACTOR, THAT IS, (ΔAF)÷AF IN PERCENT, FOR ABSOLUTE UNCERTAINTIES (A.U.) IN "ΔH", I.E. COLUMNS, OR IN "N", I.E. ROWS, IS:

A.U.(ΔH)=		.010	.020	.050	.100	.250
A.U.(N)=	.10	14.1	19.1	34.0	58.9	133.5
	.20	23.3	28.3	43.2	68.1	142.7
	.30	32.5	37.4	52.4	77.2	151.9
	.50	50.8	55.8	70.7	95.6	170.2
	1.00	96.6	101.6	116.5	141.4	216.0

Table 8

ENTER AS A VECTOR THE ABSOLUTE UNCERTAINTY IN N:

⎕:

.1 .2 .3 .5 1

ENTER AS A VECTOR THE ABSOLUTE UNCERTAINTY IN ΔH:

⎕:

.01 .02 .05 .1 .25

ENTER THE FIELD AND STRESS TEMPERATURES (∘C) AS A VECTOR:

⎕:

85 150

ENTER THE RATIO OF STRESS TO FIELD CURRENT DENSITIES:

⎕:

5

THE MATRIX OF RELATIVE UNCERTAINTIES OF THE ACCELERATION FACTOR, THAT IS, (ΔAF)÷AF IN PERCENT, FOR ABSOLUTE UNCERTAINTIES (A.U.) IN "ΔH", I.E. COLUMNS, OR IN "N", I.E. ROWS, IS:

A.U.(ΔH)=		.010	.020	.050	.100	.250
A.U.(N)=	.10	21.1	26.0	41.0	65.8	140.5
	.20	37.2	42.1	57.1	81.9	156.6
	.30	53.3	58.2	73.2	98.0	172.7
	.50	85.4	90.4	105.3	130.2	204.9
	1.00	165.9	170.9	185.8	210.7	285.3

Table 9

ENTER AS A VECTOR THE ABSOLUTE UNCERTAINTY IN N:

⎕:

.1 .2 .3 .5 1

ENTER AS A VECTOR THE ABSOLUTE UNCERTAINTY IN ΔH:

⎕:

.01 .02 .05 .1 .25

ENTER THE FIELD AND STRESS TEMPERATURES (∘C) AS A VECTOR:

⎕:

85 150

ENTER THE RATIO OF STRESS TO FIELD CURRENT DENSITIES:

⎕:

10

THE MATRIX OF RELATIVE UNCERTAINTIES OF THE ACCELERATION FACTOR, THAT IS, (ΔAF)÷AF IN PERCENT, FOR ABSOLUTE UNCERTAINTIES (A.U.) IN "ΔH", I.E. COLUMNS, OR IN "N", I.E. ROWS, IS:

A.U.(ΔH)=		.010	.020	.050	.100	.250
A.U.(N)=	.10	28.0	33.0	47.9	72.8	147.4
	.20	51.0	56.0	70.9	95.8	170.4
	.30	74.1	79.0	94.0	118.8	193.5
	.50	120.1	125.1	140.0	164.9	239.5
	1.00	235.2	240.2	255.1	280.0	354.6

Table 10

```
ENTER AS A VECTOR THE ABSOLUTE UNCERTAINTY IN N:
⎕:
      .1 .2 .3 .5 1
ENTER AS A VECTOR THE ABSOLUTE UNCERTAINTY IN ΔH:
⎕:
      .01 .02 .05 .1 .25
ENTER THE FIELD AND STRESS TEMPERATURES (•C) AS A VECTOR:
⎕:
      50 125
ENTER THE RATIO OF STRESS TO FIELD CURRENT DENSITIES:
⎕:
      5
```

THE MATRIX OF RELATIVE UNCERTAINTIES OF THE ACCELERATION FACTOR,
THAT IS, (ΔAF)÷AF IN PERCENT, FOR ABSOLUTE UNCERTAINTIES (A.U.)
IN "ΔH", I.E. COLUMNS, OR IN "N", I.E. ROWS, IS:

A.U.(ΔH)=		.010	.020	.050	.100	.250
A.U.(N)=	.10	22.9	29.6	49.9	83.7	185.1
	.20	39.0	45.7	66.0	99.8	201.2
	.30	55.0	61.8	82.1	115.9	217.3
	.50	87.2	94.0	114.3	148.1	249.5
	1.00	167.7	174.5	194.8	228.6	330.0

Table 11

ENTER AS A VECTOR THE ABSOLUTE UNCERTAINTY IN N:

⎕:

    .1 .2 .3 .5 1

ENTER AS A VECTOR THE ABSOLUTE UNCERTAINTY IN ΔH:

⎕:

    .01 .02 .05 .1 .25

ENTER THE FIELD AND STRESS TEMPERATURES (°C) AS A VECTOR:

⎕:

    50 125

ENTER THE RATIO OF STRESS TO FIELD CURRENT DENSITIES:

⎕:

    100

THE MATRIX OF RELATIVE UNCERTAINTIES OF THE ACCELERATION FACTOR, THAT IS, (ΔAF)÷AF IN PERCENT, FOR ABSOLUTE UNCERTAINTIES (A.U.) IN "ΔH", I.E. COLUMNS, OR IN "N", I.E. ROWS, IS:

A.U.(ΔH)=		.010	.020	.050	.100	.250
A.U.(N)=	.10	52.8	59.6	79.9	113.7	215.1
	.20	98.9	105.6	125.9	159.7	261.2
	.30	144.9	151.7	172.0	205.8	307.2
	.50	237.0	243.8	264.1	297.9	399.3
	1.00	467.3	474.0	494.3	528.1	629.6